Generalized Planck Scales and Possible Cosmological Consequences

Cenek Kodejska

Abstract. We present a new set of the fundamental constants of the nature which are obtained from other fundamental constants such as $\{h, c, G, \alpha\}$ and from the new low called Generalized Planck Scales. These new constants are in a remarkable accordance with the characteristics of our observable universe (mass, horizon radius, age of the universe, matter density). Possible cosmological consequences like the value of Hubble constant, age of the universe or the ratio between matter density and Λ - density are discussed.

1. Introduction

In the 1870's G.J. Stoney, the Irish physicist and the first who measured the value of elementary charge *e* and introduced into physics the term "electron", constructed from *e*, *c*, *G* universal units with dimensions of length, time and mass: $l_S = e (G)^{1/2}/c^2$, $t_S = e (G)^{1/2}/c^3$, $m_S = e / (G)^{1/2}$. The expression for m_S has been derived by equating the Coulomb and Newton forces, [3].

When M. Planck discovered in 1899 h, he introduced as universal units of Nature for basic entities of space, time and matter length $l_P = h / m_P c$, time $t_P = h / m_P c^2$ and mass $m_P = (hc/G)^{1/2}$.

Two century afterwards, Planck Scale Phenomena invites physicist of the whole world. The crucial question is: is there any physical significance to these natural units? Quantum gravity research largely takes for granted a positive answer to this question: Planck units present the physical scale of features relevant to a theory of quantum gravity and appropriate processes, [7].

In the following we show, that Planck scales as well as Stoney units are only one of many potential cases of generalized Planck Scales. Possible cosmological consequences, such as the age of the universe, Λ - density or the ratio between Ω_{Λ} and Ω_{M} are discussed.

2. Generalized Planck Scales – GPS

We accepted an other approach to the derivation of dimensional constants such as mass, length and time than [1]-[8]. Firstly, we extend a number of fundamental constants from {h, c, G} to {h, c, G, e, μ_0 } because we believe that the electrodynamics (characterized by the value of elementary charge e and by magnetic constant μ_0) has to be involved in our considerations as well as the quantum of action h, c as the basis of relativity theory and the Newton gravitational coupling constant G as ever-present gravity. The values of the constants have been adopted from []. We show that such units of length, time and mass as Planck's, Stoney's , Schrodinger's or Dirac's are only special cases of the generalized ones.

If we take into account that a set of the fundamental constants {*h*, *c*, *G*, *e*, μ_0 } have dimensions (in SI units):

$$\begin{split} & [h] = kg \cdot m^2 \cdot s^{-1}, \\ & [c] = m \cdot s^{-1}, \\ & [G] = kg^{-1} \cdot m^3 \cdot s^{-2}, \\ & [e] = s \cdot A, \\ & [\mu_0] = kg \cdot m \cdot s^{-2} \cdot A^{-2}, \end{split}$$

we can linearize power of these dimensions and write a linear system of equations in the matrix form as follows:

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ 2 & 1 & 3 & 0 & 1 \\ -1 & -1 & -2 & 1 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a_1 & a_1 & a_1 & a_1 \\ a_2 & a_2 & a_2 & a_2 \\ a_3 & a_3 & a_3 & a_3 \\ a_4 & a_4 & a_4 & a_4 \\ a_5 & a_5 & a_5 & a_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
(1),

or simply

 $C \cdot A = E,$

where *C* is matrix created from power of dimensions of the fundamental constants (first column is *h*, second *c*, third *G*, etc.), *A* is matrix composed by some real coefficients $\{a_1, a_2, a_3, a_4, a_5\}$ and *E* is identity matrix.

Solving these equations (1), we can find for mass M (first column of E_4), length L (second column of E_4), time T (third column of E_4) and current I (fourth column of E_4) following parametric solutions:

for *M*:

which means that we have a general parametric solution for a set of real parameters { a_1 , a_2 , a_3 , a_4 , a_5 } in a form

$$\left[\frac{1}{2}-a_5;\frac{1}{2}+a_5;-\frac{1}{2};2a_5;a_5\right],$$

and thus in general for mass-scale we get (for simplicity we replace a_5 by κ)

$$m(\kappa) = h^{\frac{1}{2}-\kappa} \cdot c^{\frac{1}{2}+\kappa} \cdot G^{-\frac{1}{2}} \cdot e^{2\kappa} \cdot \mu_0^{\kappa}.$$
(2)

Accordingly for length *L* we have:

1	0	-1	0	1 0	1	0	-1	0	1 0	1	0	-1	0	1 0
2	1	3	0	1 1	0	1	5	0	-11	0	1	5	0	-11
-1	-1	-2	1	-20^{2}	0	-1	-3	1	-10^{\sim}	0	0	2	1	-2 1
0	0	0	1	-20	0	0	0	1	-20	0	0	0	1	-2 0

$$\left[\frac{1}{2}-a_{5}; a_{5}-\frac{3}{2}; \frac{1}{2}; 2a_{5}; a_{5}\right],$$

$$l(\kappa) = h^{\frac{1}{2}-\kappa} \cdot c^{\kappa-\frac{3}{2}} \cdot G^{\frac{1}{2}} \cdot e^{2\kappa} \cdot \mu_0^{\kappa}, \qquad (3)$$

for time *T*:

and finally for current *I*:

$$\left[-\frac{1}{2}-a_5; a_5+\frac{5}{2}; -\frac{1}{2}; 2a_5+1; a_5\right],\$$

$$I(\kappa) = h^{-\frac{1}{2}-\kappa} \cdot c^{\kappa+\frac{5}{2}} \cdot G^{-\frac{1}{2}} \cdot e^{2\kappa+1} \cdot \mu_0^{\kappa}.$$
 (5)

We still modify equations (1) - (5) to get the best form as follows:

$$m(\kappa) = \sqrt{\frac{hc}{G}} \left(\frac{\mu_0 c e^2}{h}\right)^{\kappa} = m_P \cdot (2\alpha)^{\kappa}, \qquad (6)$$

$$l(\kappa) = \sqrt{\frac{hG}{c^3}} \left(\frac{\mu_0 c e^2}{h}\right)^{\kappa} = l_P \cdot \left(2\alpha\right)^{\kappa}, \qquad (7)$$

$$t(\kappa) = \sqrt{\frac{hG}{c^5}} \left(\frac{\mu_0 c e^2}{h}\right)^{\kappa} = t_P \cdot (2\alpha)^{\kappa}, \qquad (8)$$

$$I(\kappa) = \sqrt{\frac{c^5}{Gh}} e\left(\frac{\mu_0 c e^2}{h}\right)^{\kappa} = I_P \cdot (2\alpha)^{\kappa}, \qquad (9)$$

or in a logarithmic form

$$\ln\left(\frac{m(\kappa)}{m_{P}}\right) = \kappa \cdot \ln(2\alpha), \qquad (10)$$

$$\ln\left(\frac{l(\kappa)}{l_{P}}\right) = \kappa \cdot \ln(2\alpha), \qquad (11)$$

$$\ln\left(\frac{t(\kappa)}{t_p}\right) = \kappa \cdot \ln(2\alpha), \qquad (12)$$

$$\ln\left(\frac{I(\kappa)}{I_{p}}\right) = \kappa \cdot \ln(2\alpha), \qquad (13)$$

where m_{P, l_P, t_P, l_P} are Planck mass, Planck length, Planck time and Planck current, α is the fine structure constant and κ is so-called the *quantum coefficient* (this name is has been chosen for reasons that this coefficient can play an important role in a quark-lepton model).

For our next consideration we will continue only with relationships for $m(\kappa)$, $l(\kappa)$ and $t(\kappa)$. We can see that for $\kappa = 0$ we get standard Planck scales with no reduction of the value of *h*,

$$m(0) = \sqrt{\frac{hc}{G}} = m_p = 5.455552 \cdot 10^{-8} \ kg$$

$$l(0) = \sqrt{\frac{hG}{c^3}} = l_p = 4.051319 \cdot 10^{-35} \ m$$

$$t(0) = \sqrt{\frac{hG}{c^5}} = t_p = 1.351375 \cdot 10^{-43} \ s$$

$$(14)$$

and for $\kappa = \frac{1}{2}$ Stoney scales (these are independent of *h*)

$$m(1/2) = \sqrt{\frac{hc}{G}} \left(\frac{\mu_0 c e^2}{h}\right)^{1/2} = \sqrt{\frac{\mu_0}{G}} c e = m_s$$

$$l(1/2) = \sqrt{\frac{hG}{c^3}} \left(\frac{\mu_0 c e^2}{h}\right)^{1/2} = \sqrt{\mu_0 G} \frac{e}{c} = l_s$$

$$t(1/2) = \sqrt{\frac{hG}{c^5}} \left(\frac{\mu_0 c e^2}{h}\right)^{1/2} = \sqrt{\mu_0 G} \frac{e}{c^2} = t_s$$
(15)

Because other units (such as Schrodinger, Hartree-Bohr, Dirac or QED-Stille units) can be obtained from Planck and Stoney units with the certain substitution [1], we can denominate scales described by Equ. (6) - (9) as the *main scales* and others as the *derived* ones.

Note, that Stoney mass, length and time in equations (15) differs from units defined by Stoney [1], [3] by the constant $(1/4\pi)^{1/2}$ because of the fact, that the expression for m_S in Stoney article has been derived by equating the Coulomb and Newton forces. This discrepancy can be corrected by using \hbar instead of h, which leads to the analogous result as follows:

$$m(\kappa) = \sqrt{\frac{\hbar c}{G}} \left(\frac{\mu_0 c e^2}{2h}\right)^{\kappa} = m_{P_{red}} \cdot (\alpha)^{\kappa}, \qquad (16)$$

$$l(\kappa) = \sqrt{\frac{\hbar G}{c^3}} \left(\frac{\mu_0 c e^2}{2h}\right)^{\kappa} = l_{P_{red}} \cdot (\alpha)^{\kappa}, \qquad (17)$$

$$t(\kappa) = \sqrt{\frac{\hbar G}{c^5}} \left(\frac{\mu_0 c e^2}{2h}\right)^{\kappa} = t_{P_{red}} \cdot (\alpha)^{\kappa}.$$
(18)

3. Physical Meaning of GPS

As we can see from the equations (6) - (8) or (16) - (18), these formulae can be interpreted as a power low for arbitrary mass, length and time (from elementary particles to the whole universe) which is only created from the fundamental constants of nature such as {*h*, *c*, *G*, *e*, μ_{0} , α } and some real number κ called the *quantum coefficient*.

The relatively close range for a value of κ can be estimated as an interval $\langle -33; 33 \rangle$ where (-33) corresponds to the whole universe (mass as well as length or time) and right side of an interval, (33) is a lower limit for masses of the elementary particles, for example.

The value of the quantum coefficient κ is illustrated on the Fig.1.



FIG.1: Quantum coefficient κ in relation to rough masses of the objects

We behold that the main significance of GPS is in a coupling between the world of the elementary particles and macroscopic objects including our universe through a very close interval for the quantum coefficient.

4. New Universal Constants of the Nature and Cosmological Consequences

What are fundamental constants of the nature? We have a set of the physical constants such as $\{h, c, G\}$ or $\{h, c, G, e, \mu_0\}$. However, there are other fundamental constants as $\pi = 3.1415926$, e = 2.71828 or $\varphi = 1.6180339$ (Golden Section) which are fundamental mathematical ones. These constants are present in many laws of physics or in a nature itself [9].

Now we define set of the new universal constants, which are related to GPS and the size of these can have possible cosmological implications.

Earliest we consider new scales for $\kappa = -\frac{1}{2}$:

$$m(-1/2) = \sqrt{\frac{hc}{G}} \left(\frac{\mu_0 c e^2}{h}\right)^{-1/2} = \sqrt{\frac{1}{\mu_0 G}} \frac{h}{e} = m_K = 4.515866 \cdot 10^{-7} kg , \qquad (19)$$

$$l(-1/2) = \sqrt{\frac{hG}{c^3}} \left(\frac{\mu_0 c e^2}{h}\right)^{-1/2} = \sqrt{\frac{G}{\mu_0}} \frac{h}{ec^2} = l_K = 3.353504 \cdot 10^{-34} m, \qquad (20)$$

$$t(-1/2) = \sqrt{\frac{hG}{c^5}} \left(\frac{\mu_0 c e^2}{h}\right)^{-1/2} = \sqrt{\frac{G}{\mu_0}} \frac{h}{ec^3} = t_K = 1.118609 \cdot 10^{-42} s, (21)$$

$$\rho(-1/2) = \frac{m_K}{l_K^3} = \frac{\mu_0 \ c \ e^2}{h} \frac{c^5}{G^2 h} = 2\alpha \ \rho_P = \rho_K = 1.197415 \cdot 10^{94} \ kg \ \cdot m^{-3}.$$
 (22)

In the following we set new universal constants as:

$$M_{K} = m_{K} \cdot \exp[\alpha^{-1}] = 1.474753 \cdot 10^{53} kg , \qquad (23)$$

$$L_{K} = l_{K} \cdot \exp[\alpha^{-1}] = 1.095159 \cdot 10^{26} m, \qquad (24)$$

$$T_{K} = t_{K} \cdot \exp[\alpha^{-1}] = 3.653058 \cdot 10^{17} \, s = 11.58 \, Gyr \,, \tag{25}$$

$$P_{K} = \frac{M_{K}}{L_{K}^{3}} = \rho_{K} \cdot \exp[-2\alpha^{-1}] = 1.122762 \cdot 10^{-25} \ kg \cdot m^{-3}, \tag{26}$$

where α is the fine structure constant and $m_{K_i} l_{K_i} t_{K_i} \rho_K$ are specified above. Here and hereinafter by reason of the possible mistake of e = 2.71828 and e (elementary charge) we will use the expression for mathematical constant e^x as exp[x].

The values of these constants, M_K , L_K , T_K , P_K , are in remarkable accordance with the characteristics of our observable universe. Now we dissect these values concerning articles [10], [11], [14], [15] and present-day observations, such as the current value of the cosmological constant, Hubble parameter or critical density.

A. Hubble Constant and Age of the Universe

Primarily we focus on the constant T_K . Consider "Sandage Consistency Test" [12] where $H_0t_0 = 0.94 \pm 0.14 \approx 1$ ($H_0t_0 = 2/3$ is inconsistent with current data). If we take a value of T_K from Equ.(25) as the age of the Universe, we get

$$H_0 = 1/T_K = 84.47 \ km \cdot s^{-1} \cdot Mpc^{-1}.$$
 (27)

Note, that this value is not obtained by observations, but is calculated only from the fundamental constants of nature! The result is in perfect harmony with measurements of the Hubble constant using the Cepheid variables in the Virgo cluster and the relative distance between the Virgo and the Coma cluster, which yield $H_0 = 87 \pm 7$ km s⁻¹ Mpc⁻¹, [20], $H_0 = 80 \pm 17$ km·s⁻¹·Mpc⁻¹, [21] or $H_0 = 84 \pm 8$ km·s⁻¹·Mpc⁻¹, [17]. These observations disagree with the other HST measurements [22], of $H_0 = 58 \pm 4$ km·s⁻¹·Mpc⁻¹ or with recent value $H_0 = 62.3 \pm 1.3$ km·s⁻¹·Mpc⁻¹, [18], [23].

To lower the value taking out from Equ.(27) we can use the method mentioned for example in [10], [11], [16] or in [24]. We accepted the practice suggested by Tully [10] and illustrated in [11].

Imagine a local mass concentration M superimposed on a Hubble flow. At a distance R from the mass, the radially outward velocity V may be given by

$$V = -\left(\frac{2GM}{R}\right)^{1/2} + HR \equiv H_{eff}R.$$
(28)

The first term is an inward velocity corresponding to a zero value at infinity, while the second term is the Hubble flow with the Hubble parameter H. We can rewrite Equ.(28) as

$$H_{eff} = H_{local} = H - \left(\frac{2GM}{R^3}\right)^{1/2},$$
 (29)

where effective (or local) Hubble constant is smaller than the true Hubble constant H, closer to the mass concentration. As we go away from M, the local value of local Hubble parameter approaches the true value.

Tully estimates that the local anomaly may be caused by a mass of the order $10^{14} \text{ M}^{\circ} - 10^{15} \text{ M}^{\circ}$ in the Virgo Cluster. If we take the mean value for a mass $M = 5 \cdot 10^{14} \text{ M}^{\circ}$ and the mean distance R = 20 Mpc, [25], we get

$$H_{eff} = 61.26 \ km \cdot s^{-1} \cdot Mpc^{-1} = 15.96 \ Gyr.$$
(30)

This calculated effective Hubble constant is surprisingly close to the current data ($H_0 = 62.3$) mentioned above. Analogous result we can realize from the consideration that local Hubble constant is of 1.33 times larger than global one for a present matter density $\Omega_M = 0.2$, [16].

Our H_{eff} than yields

$$H_{eff} = 63.52 \ km \cdot s^{-1} \cdot Mpc^{-1} = 15.4 \ Gyr \,, \qquad (31)$$

which is keeping value $H_0 = (62.3 \pm 1.3) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$.

The second test for our theory of new universal constants comes from recently published considerations about the Hubble flow around the Cen A/M 83 galaxy complex, [13].

In paper [13] HST/ACS images, color-magnitude diagrams for 24 nearby galaxies in and near the constellation of Centaurus and mass estimates of the complex Cen A/M 83 are presented. The mass has been derived for a flat cosmological model with non-zero Λ - term as

$$M_{T} = \frac{\pi^{2}}{8G} \cdot R_{0}^{3} \cdot T_{0}^{-2} \cdot f(\Omega_{M})^{-2}, \qquad (32)$$

where

$$f(\Omega_M) = \frac{1}{1 - \Omega_M} - \frac{1}{2} \Omega_M \cdot (1 - \Omega_M)^{-3/2} \cdot \arccos h[(2/\Omega_M) - 1],$$
(33)

 M_T is the total mass of a group, R_0 is the turn-over radius and T_0 the age of the universe, [13].

If we arrange Equ.(32), we can write

$$\frac{8G}{\pi^2} \cdot \frac{M_T}{R_0^3} \cdot T_0^2 = f(\Omega_M)^{-2},$$
(34)

where the second term on the left side represents the average matter density of the Cen A/M 83 complex. On the assumption that average matter density is roughly the same in the whole universe, we can apply the new universal constant P_K to get the age of the universe. Thus, we have

$$T_0 = \sqrt{\frac{\pi^2}{8G} \cdot \frac{1}{P_K} \cdot \frac{1}{f(\Omega_M)^2}}.$$
(35)

For $\Omega_M = 0.3$, we have f(0.3) = 0.808 and with $P_K = 1.1233195 \cdot 10^{-25} kg \cdot m^{-3}$ this yields

$$T_0 = 5.0155215 \cdot 10^{17} \, s = 15.9 \, Gyr$$

$$H_0 = 1/T_0 = 61.52 \, km \cdot s^{-1} \cdot Mpc^{-1}.$$
 (36)

Surprisingly, we have practically the same result as that obtained in other way mentioned above.

By reason of the results in the next section, we mention the results for $\Omega_M = 0.39$, concretely:

$$T_0 = 5.1922341 \cdot 10^{17} s = 16.46 Gyr$$

$$H_0 = 1/T_0 = 59.42 \ km \cdot s^{-1} \cdot Mpc^{-1}$$
(37)

B. Cosmological Constant and the A-density ρ_A

The value of the cosmological constant derived from vacuum fluctuation has been discussed recently by Gurzadyan and Xue [14], [15], Djorgovski and Gurzadyan[26] or by Padmanabhan [27]. As shown in [14], one can get for the dark energy density

$$\Lambda_{\rho} = \frac{1}{2} \frac{\hbar c \pi}{a^4} N_{\text{max}} \left(N_{\text{max}} + 1 \right) = \frac{\hbar c \pi}{2a^2 l_P^2} , \qquad (38)$$

where $N_{max} = a / l_P$ is the maximum number of relevant modes of the vacuum fluctuations, l_P is Planck length and the present characteristic size of the Universe $a \approx 10^{26}$ m. We can express this result as an effective matter density, [15]

$$\rho_{\Lambda} = \frac{\Lambda_{\rho}}{c^2} = \frac{\hbar\pi}{2c} \frac{1}{a^2 l_p^2} kg \cdot m^{-3}.$$
 (39)

If we replace in the foregoing relationship the value of L_K instead of a, we get

$$\rho_{\Lambda} = \frac{\Lambda_{\rho}}{c^2} = \frac{\hbar\pi}{2c} \frac{1}{L_{\kappa}^2 l_{\rho}^2} = 1.7640893 \cdot 10^{-25} \ kg \cdot m^{-3}. \tag{40}$$

Compare this value now with the value P_K calculated in Equ. (26). If we interpret P_K as an present matter density of the Universe, than the rate of $\rho_{\Lambda} / P_K = 1.57$ which yields (on the assumption that $\Omega_{\Lambda} + \Omega_M = 1$)

$$\frac{\Omega_{\Lambda}}{\Omega_{M}} = \frac{0.61}{0.39}.$$
(41)

This result is in accordance with observed data as well as with theoretical limits following from the different theories, for example [19], where one can find the limit for Ω_M as $0.1 < \Omega_M < 0.4$ and $\Omega_A < 0.7$ as an upper limit in a flat universe.

Remember again that the values for $\Omega_A = 0.61$ and $\Omega_M = 0.39$ were obtained by the calculation purely from the fundamental constants of nature.

Finally, we note that in the case of the definition M, L, T with m_P , l_P and t_P instead of m_K , l_K and t_K the values of H_0 as well as ρ_A / ρ_m are out of the range of the present observable one.

5. Conclusion

We defined two new sets of the fundamental constants, $\{m_K, l_K, t_K, \rho_K\}$, $\{M_K, L_K, T_K, P_K\}$, where the first follows from generalized Planck scales which allow to construct with help a quantum coefficient in the interval $\langle -33 \rangle$; $33 \rangle$ for instance arbitrary mass in the range from the elementary particles to the whole Universe. The second provides comparable values with the present characteristics of the Universe and leads to the ratio between matter density parameter Ω_M and Λ -density parameter Ω_Λ with a very good result $\Omega_\Lambda = 0.61$ and $\Omega_M = 0.39$ obtained entirely by virtue of the calculation from the fundamental constants $\{h, c, G, \alpha, \exp[\alpha^{-1}]\}$.

We retrieved by many different manners (based only on fundamental constants) for the Hubble constant value $H_0 = (61.43 \pm 1.68) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ and the age of the universe is then determined as $T_0 = 1/H_0 = (15.93 \pm 0.43)$ Gyr.

To summarize all values taking out only from the computations using the set of the fundamental constants $\{h, c, G, \alpha\}$, all from these are in excellent correspondence with the observable data as well as with other theoretical predictions.

References

- [1] M. J. Duff, Comment on time-variation of fundamental constants, hep-th/0208093, (2004)
- [2] F. I. Cooperstock, Extended Planck Scale, hep-th/0302080, (2003)
- [3] M. J. Duff, L. B. Okun and G. Veneziano, *Trialogue on the number of fundamental constants*, physics/0110060, (2002)
- [4] D. Meschini, Planck-scale physics, gr-qc/0601097, (2006)
- [5] F.Pisano and N. O. Reis, *Natural units, numbers and numerical clusters*, hep-ph/0112097 (2001)
- [6] B. G. Sidharth, *Planck Scale Phenomena*, physics/0208045, (2002)
- [7] F. I. Cooperstock and V. Faraoni, *The New Planck Scale: Quantized Spin and Charge Coupled to Gravity*, gr-qc/0305092, (2003)
- [8] B. G. Sidharth, The Emergence of the Planck Scale, physics/0002035, (2000)
- [9] M. Livio, The Golden Ratio, Broadway Books, New York 2002
- [10] P. J. E. Peebles, S. D. Phelps, E. J. Shaya, R. B. Tully, *Radial and transverse velocities* of nearby galaxies, astro-ph/0010480, (2000)
- [11] J. V. Narlikar, Introduction to cosmology, Cambridge University Press 1993, p.303
- [12] M. S. Turner, Time at the Beginning, astro-ph/0106262, (2001)
- [13] I. D. Karachentsev, R. B. Tully et al., *The Hubble flow around the Cen A/M 83 galaxy complex*, astro-ph/0603091, (2006)
- [14] V. G. Gurzadyan and She-Sheng Xue, *On the estimation of the current value of the cosmological constant*, astro-ph/0105245, (2003)
- [15] V. G. Gurzadyan and She-Sheng Xue, On the vacuum fluctuations and the cosmological

constant: Comment on the paper by T.Padmanabhan, astro-ph/0510459, (2006)

- [16] Xiang-Ping Wu et al., *On the measurement of the Hubble constant in a local low-density universe*, ApJ, 448, (1995)
- [17] R. B. Tully, *The cosmological parameters* H_0 and Ω_0 , astro-ph/9802026, (1998)
- [18] G. A. Tammann, *The Ups and Downs of the Hubble Constant*, Modern Astronomy 19, (2006)
- [19] S. M. Carroll, The Cosmological Constant, astro-ph/0004075, (2000)
- [20] M. J. Pierce et al., Nature, 371, 385, (1994)
- [21] W. L. Friedman et al., *Nature 371*, 757, (1994)
- [22] A. Sandage, A. Saha at al., *Cepheid Calibration of the Peak Brightness of type Ia* Supernovae, ApJ 460, L15-L18, (1996)
- [23] A. Sandage, G. A. Tammann, A. Saha, B. Reindl, F. D. Macchetto and N. Panagia, The Hubble Constant: A Summary of the HST Program for the Luminosity Calibration of Type Ia Supernovae by means of Cepheids, astro-ph/0603647, (2006)
- [24] R. Mohayaee, R. B. Tully and U. Frisch, *Reconstruction of large-scale peculiar velocity fields*, astro-ph/0410063, (2004)
- [25] A. Sandage and G. A. Tammann, *The Distance to the Virgo Cluster from a Recalibrated Tully-Fisher Relation*, astro-ph/0608677, (2006)
- [26] S. G. Djorgovski and V. G. Gurzadyan, Dark Energy from Vacuum Fluctuations, astro-ph/0610204, (2006)
- [27] T. Padmanabhan, Dark Energy: Mystery of the Millennium, astro-ph/0603114, (2006)
- [28] P. J. Mohr and B. N. Taylor, *CODATA values of the fundamental constants*, Rev. Mod. Phys., Vol. 77, No.1 (2005)