20a) MOCNINY

Věty pro počítání s mocninami s celým mocnitelem (exponentem).

Pro každá dvě reálná čísla a, b a pro libovolná celá čísla r, s platí:

$a^r\cdot a^s=a^{r+s}$	$(a^r)^s=a^{rs}$	$a^r:a^s=a^{r-s}\ (a\neq0)$
$(a\cdot b)^r=a^r\cdot b^r$	$\left(\frac{a}{b}\right)^r=\frac{a^r}{b^r}\ (b\neq0$	

Dále platí:

$\boxed{a^0=1\ ;\ (a\in R-\{0\}}$

$\boxed{a^{-m}=\frac{1}{a^m}=\left(\frac{1}{a}\right)^m\ ;\ a\neq0,\ m\in Z^-}$

$\boxed{\left(\frac{a}{b}\right)^{-2}=\left(\frac{b}{a}\right)^2\ ;\ a,b\in R-\{0\}}$

Příklady 1 – 7 jsou ze sbírky úloh pro OA

Příklad 1: Vypočítejte:

a) $\frac{2^{-2}\cdot2^2}{2^{-3}}=2^{-2}\cdot2^2\cdot2^3=2^{(-2+2+3)}=\boxed{2^3\,(8)}$

b) $\frac{5^{-2}\cdot2^3}{(2\cdot5)^{-4}}=5^{-2}\cdot2^3\cdot(2\cdot5)^4=5^{-2}\cdot2^3\cdot2^4\cdot5^4=2^{(3+4)}\cdot5^{(-2+4)}=\boxed{2^7\cdot5^2}$

c) $\frac{2^2\cdot3^{-1}}{(2\cdot3)^{-1}}=2^2\cdot3^{-1}\cdot(2\cdot3)^1=2^2\cdot3^{-1}\cdot2^1\cdot3^1=2^3\cdot3^0=2^3\cdot1=\boxed{2^3\,(8)}$

d) $\frac{3^3\cdot2^{-3}}{6^{-1}}=3^3\cdot2^{-3}\cdot6=3^3\cdot2^{-3}\cdot2^1\cdot3^1=\boxed{2^{-2}\cdot3^4}$

e) $\frac{7^{-1}\cdot3^{-5}}{21^2}=7^{-1}\cdot3^{-5}\cdot(21)^{-2}=7^{-1}\cdot3^{-5}\cdot(3\cdot7)^{-2}=7^{-1}\cdot3^{-5}\cdot3^{-2}\cdot7^{-2}=\boxed{3^{-7}\cdot7^{-3}}$

f) $\frac{12^{-1}\cdot4^2}{3^{-3}}=(2\cdot2\cdot3)^{-1}\cdot(2\cdot2)^2\cdot3^3=2^{-1}\cdot2^{-1}\cdot3^{-1}\cdot2^2\cdot2^2\cdot3^3=\boxed{2^2\cdot3^2}$

Příklad 2: Vypočítejte:

a) $\left(\frac{1}{2}\right)^{-2}\cdot\left(\frac{1}{2}\right)^{-3}=\left(\frac{2}{1}\right)^2\cdot\left(\frac{2}{1}\right)^3=2^2\cdot2^3=\boxed{2^5}$

b) $\left(\frac{4}{3}\right)^{-2}\cdot\left(\frac{2}{3}\right)^{-1}=\left(\frac{3}{4}\right)^{2}\cdot\left(\frac{3}{2}\right)^{1}=\frac{3^2}{4^2}\cdot\frac{3}{2}=\frac{3^2}{(2^2)^2}\cdot\frac{3}{2}=\frac{3^2}{2^4}\cdot\frac{3}{2}=$

$=\frac{3^3}{2^5}=\boxed{3^3\cdot 2^{-5}}$

c) $\left(\frac{4}{9}\right)^{-2}\cdot\left(\frac{8}{27}\right)^{3}=\left(\frac{9}{4}\right)^{2}\cdot\left(\frac{8}{27}\right)^{3}=\frac{(3^2)^2}{(2^2)^2}\cdot\frac{(2^3)^3}{(3^3)^3}=\frac{3^4}{2^4}\cdot\frac{2^9}{3^9}$

$=3^4\cdot2^{-4}\cdot2^9\cdot3^{-9}=\boxed{2^5\cdot3^{-5}}$

d) $\left(\frac{2}{5}\right)^{-2}\cdot\left(\frac{25}{4}\right)^{-1}=\left(\frac{5}{2}\right)^{2}\cdot\left(\frac{4}{25}\right)^{1}=\frac{5^2}{2^2}\cdot\frac{2^2}{5^2}=1$

Příklad 3:

a) $\frac{12^{-1}\cdot4^{-3}}{9^3}=(2.2.3)^{-1}\cdot(2.2)^{-3}\cdot(3.3)^{-3}=2^{-1}\cdot2^{-1}\cdot3^{-1}\cdot2^{-3}\cdot2^{-3}\cdot3^{-3}\cdot3^{-3}=\boxed{2^{-8}\cdot3^{-7}}$

b) $\frac{22^{-1}\cdot11^3}{4^{-2}\cdot2}=(2.11)^{-1}\cdot11^3\cdot(2.2)^2\cdot2^{-1}=2^{-1}\cdot11^{-1}\cdot11^3\cdot2^2\cdot2^2\cdot2^{-1}=\boxed{2^2\cdot11^2}$

c) $\frac{9^{-2}\cdot45^{-3}\cdot3^2}{25^{-1}}=(3.3)^{-2}\cdot(3.3.5)^{-3}\cdot3^2\cdot(5.5)^1=3^{-2}\cdot3^{-2}\cdot3^{-3}\cdot3^{-3}\cdot5^{-3}\cdot3^2\cdot5^1\cdot5^1=$

$=\boxed{5^{-1}\cdot3^{-8}}$

d) $\frac{49^{-3}\cdot6^{-3}}{14^{-2}\cdot9^{-2}}=(7.7)^{-3}\cdot(2.3)^{-3}\cdot(2.7)^2\cdot(3.3)^2=7^{-3}\cdot7^{-3}\cdot2^{-3}\cdot3^{-3}\cdot2^2\cdot7^2\cdot3^2\cdot3^2=$

$=\boxed{2^{-1}\cdot3\cdot7^{-4}}$

Příklad 4: Vypočítejte za předpokladu, že jsou všechna čísla různá od nuly.

a) $\frac{x^2y^{-2}z^3}{(xyz)^{-2}}=x^2y^{-2}z^3\cdot(xyz)^2=x^2y^{-2}z^3\cdot x^2\cdot y^2\cdot z^2=x^4y^0z^5=\boxed{x^4z^5}$

b) $\frac{\left(\frac{x}{y}\right)^{-2}\cdot\left(\frac{y}{x}\right)^{3}}{(xy)^{-4}}=\frac{\left(\frac{y}{x}\right)^{2}\cdot\left(\frac{y}{x}\right)^{3}}{(xy)^{-4}}=\left(\frac{y}{x}\right)^{5}\cdot(xy)^4=\frac{y^5}{x^5}\cdot x^4\cdot y^4=y^5\cdot x^{-5}\cdot x^4\cdot y^4=$

$=\boxed{x^{-1}\cdot y^9}$

c) $\dfrac{3x^2y^{-1}}{5xy} : \dfrac{25x^2y^{-2}}{9x} = \dfrac{3x^2}{5xy^2} : \dfrac{25x^2}{9xy^2} = \dfrac{3x^2}{5xy} \cdot \dfrac{9xy^2}{25x^2} =$

$= \dfrac{27x^3y^2}{125x^3y^2} = \boxed{\dfrac{27}{125}}$

---

d) $\left(\dfrac{2xy^{-1}}{3x^2y}\right)^{-1} : \dfrac{(3xy)^{-2}}{x^3y^2} = \left(\dfrac{3x^2y}{2xy^{-1}}\right)^{1} \cdot \dfrac{x^3y^2}{(3xy)^{-2}} = \dfrac{3}{2}xy^2 \cdot x^3y^2 \cdot (3xy)^2 =$

$= \dfrac{3}{2}xy^2 \cdot x^3y^2 \cdot 9x^2y^2 = \boxed{\dfrac{27}{2}x^6y^6}$

Příklad 5: Vypočítejte:

a) $\dfrac{0{,}000\,008}{2000} : \dfrac{6\,000}{0{,}000\,003} = \dfrac{8\cdot10^{-6}}{2\cdot10^3} : \dfrac{6\cdot10^3}{3\cdot10^{-6}} = \dfrac{8\cdot10^{-6}}{2\cdot10^3} \cdot \dfrac{3\cdot10^{-6}}{6\cdot10^3} =$

$= \dfrac{\overset{2}{\cancel{24}}\cdot10^{-12}}{\cancel{12}\ 10^6} = 2\cdot10^{-12}\cdot10^{-6} = \boxed{2\cdot10^{-18}}$

---

b) $\dfrac{2\cdot10^{-3}\cdot4\cdot10^5}{0{,}000\,08} : \dfrac{25\cdot10^3}{0{,}005} = \dfrac{8\cdot10^2}{8\cdot10^{-5}} \cdot \dfrac{5\cdot10^{-3}}{25\cdot10^3} = 10^7\cdot\dfrac{10^{-6}}{5} =$

$= \dfrac{10^1}{5} = \boxed{2}$

---

c) $\dfrac{24000\cdot2\cdot10^{-8}}{0{,}000\,6} \cdot \dfrac{3\cdot10^5\cdot0{,}000\,9}{27} = \dfrac{2{,}4\cdot10^4\cdot2\cdot10^{-8}}{6\cdot10^{-4}} \cdot \dfrac{3\cdot10^5\cdot9\cdot10^{-4}}{27} =$

$= \dfrac{4{,}8\cdot10^{-4}\cdot10^4}{6}\cdot10^1 = \dfrac{4{,}8}{6}\cdot10 = \dfrac{48}{6} = \boxed{8}$

---

d) $\dfrac{0{,}000\,12\cdot2000}{60\,000} \cdot \dfrac{0{,}000\,001}{80\,000} = \dfrac{1{,}2\cdot10^{-4}\cdot2\cdot10^3}{6\cdot10^4} \cdot \dfrac{1\cdot10^{-6}}{8\cdot10^4} =$

$= \dfrac{2{,}4\cdot10^{-1}\cdot10^{-4}}{6} \cdot \dfrac{10^{-6}\cdot10^{-4}}{8} = \dfrac{2{,}4}{6}\cdot10^{-5}\cdot\dfrac{1}{8}\cdot10^{-10} = \dfrac{2{,}4}{48}\cdot10^{-15} =$

$= 0{,}05\cdot10^{-15} = 5\cdot10^{-2}\cdot10^{-15} = \boxed{5\cdot10^{-17}}$

Příklad 6: Vypočítejte za předpokladu, že $x \neq \pm y$, $a \neq b$, $x \neq 0$, $y \neq 0$:

a) $$\left(\frac{x+y}{x-y}\right)^{-3} \cdot \left(\frac{x-y}{a-b}\right)^{-2} \cdot \left(\frac{x+y}{a-b}\right)^{2} = \left(\frac{x-y}{x+y}\right)^{3} \cdot \left(\frac{a-b}{x-y}\right)^{2} \cdot \left(\frac{x+y}{a-b}\right)^{2} =$$

$$\frac{(x-y)^3}{(x+y)^3} \cdot \frac{(a-b)^2}{(x-y)^2} \cdot \frac{(x+y)^2}{(a-b)^2} = \frac{x-y}{x+y} \cdot \frac{1}{1} \cdot \frac{1}{1} = \boxed{\frac{x-y}{x+y}}$$

---

b) $$\left[\left(\frac{3x^{-3}}{y^4}\right)^{-3} \cdot \left(\frac{3x^5}{y^2}\right)^{-2}\right]^{-1} = \left[\left(\frac{y^4}{3x^{-3}}\right)^{3} \cdot \left(\frac{y^2}{3x^5}\right)^{2}\right]^{-1} = \left[\left(\frac{y^{12}}{3^3x^{-9}}\right) \cdot \left(\frac{y^4}{3^2x^{10}}\right)\right]^{-1} =$$

$$= \left(\frac{y^{16}}{3^5x}\right)^{-1} = \left(\frac{3^5x}{y^{16}}\right)^{1} = \boxed{3^5xy^{-16}}$$

---

c) $$\left[\left(\frac{1}{3}\right)^{-2} \cdot \left(\frac{xy^{-1}}{a}\right)^{-3} \cdot \left(\frac{a}{x^{-1}y}\right)^{2} \cdot 3^{-2}\right]^{-1} = \left[\left(\frac{3}{1}\right)^{2} \cdot \left(\frac{a}{xy^{-1}}\right)^{3} \cdot \left(\frac{ax}{y}\right)^{2} \cdot \frac{1}{3^2}\right]^{-1} =$$

zkrát $9 \cdot \frac{1}{9} = 1$

$$= \left[\left(\frac{ay}{x}\right)^{3} \cdot \left(\frac{ax}{y}\right)^{2}\right]^{-1} = \left(\frac{a^3y^3}{x^3} \cdot \frac{a^2x^2}{y^2}\right)^{-1} = \left(\frac{a^5x^2y^3}{x^3y^2}\right)^{-1} = \left(\frac{a^5y}{x}\right)^{-1} =$$

$$= \frac{x}{a^5y} = \boxed{xy^{-1} \cdot a^{-5}}$$

Příklad 7: Vypočítejte za předpokladu, že jsou všechna čísla různá od nuly.

a) $$\left[3x^4 \cdot \left(\frac{1}{2}\right)^{-2}\right]^{2} : \left[9x^{-2}\left(\frac{3}{x}\right)^{-1}\right] = \left(3x^4 2^2\right)^2 : \left(\frac{9}{x^2} \cdot \frac{x}{3}\right) = \left(3^2x^8 2^4\right) : \left(\frac{3}{x}\right) =$$

$$= 3^2x^8 2^4 \cdot \frac{x}{3} = \boxed{2^4 . 3 . x^9}$$

Konec příkladů ze sbírky pro OA

Příklad 8: Hodnoty číselných výrazů s mocninami vyjádřete jedním číslem.

a) $$\left(2\tfrac{1}{4}\right)^{-2} : \left(1\tfrac{1}{2}\right)^{-3} = \left(\frac{9}{4}\right)^{-2} : \left(\frac{3}{2}\right)^{-3} = \left(\frac{4}{9}\right)^{2} : \left(\frac{2}{3}\right)^{3} = \frac{16}{81} : \frac{8}{27} = \frac{16}{81} \cdot \frac{27}{8} = \boxed{\frac{2}{3}}$$

---

b) $\frac{2^5\cdot5^4}{125\cdot8}\cdot(-0,4)^{-2}=\frac{2^5\cdot5^4}{5^3\cdot2^3}\cdot\left(-\frac{2}{5}\right)^{-2}=(2^5\cdot5^4\cdot5^{-3}\cdot2^{-3})\cdot\left(-\frac{5}{2}\right)^2=$

$=2^2\cdot5\cdot\frac{5^2}{2^2}=2^2\cdot5\cdot5^2\cdot2^{-2}=2^0\cdot5^3=\boxed{125}$

Příklad 9: Uvedené údaje vyjádřete v jednotkách uvedených v závorce ve tvaru $a\cdot10^n$, kde $1\leq a<10$, $n\in N$.

a) 6378 km (m), b) 250 hl (l), c) 650 MW (W), d) 27 m^3 (l)

Řešení:

a) 6378 km = 6 378 000 m = $\boxed{6,378\cdot10^6\ \text{m}}$

b) 250 hl = 25 000 l = $\boxed{2,5\cdot10^4\ \text{l}}$

c) 650 MW = $650\cdot10^6$ W = $6,5\cdot10^2\cdot10^6$ W = $\boxed{6,5\cdot10^8\ \text{W}}$

d) 27 m^3 = 27 000 dm^3 (l) = $\boxed{2,7\cdot10^4\ \text{l}}$

Příklad 10: Za předpokladu, že $a,b,c\neq0$, vypočítejte:

a) $(3a^7b^{-2}c^{-3})\cdot(4a^{-6}b^2c^{-1})=12a^1b^0c^{-4}=\boxed{\frac{12a}{c^4}\ \text{(nebo } 12ac^{-4})}$

b) $\left(\frac{1}{2}a^{-2}b^3c\right)^{-3}:(4a^4b^{-8}c^{-3})=\left[\left(\frac{1}{2}\right)^{-3}a^6b^{-9}c^{-3}\right]:(4a^4b^{-8}c^{-3})=$

$=(8a^6b^{-9}c^{-3}):(4a^4b^{-8}c^{-3})=2a^2b^{-1}c^0=\boxed{\frac{2a^2}{b}\ \text{nebo}\ 2a^2b^{-1}}$

Příklad 11: Zjednodušte:

a) $2x^{n+2}\cdot3x^{n+1}=2\cdot x^n\cdot x^2\cdot3\cdot x^n\cdot x^1=6x^{n+n}\cdot x^{2+1}=6x^{2n}\cdot x^3=\boxed{6x^{2n+3}}$

---

b) $y^{x+2}\cdot y^{x-2}=y^{x+2+x-2}=\boxed{y^{2x}}$

Příklad 12: Určete výsledek bez záporných exponentů.

a) $(a^3b^{-3}):(a^{-3}b^3)=a^{3-(-3)}\cdot b^{-3-3}=a^6b^{-6}=\frac{a^6}{b^6}=\boxed{\left(\frac{a}{b}\right)^6}$

---

b) $(u^rv^{-2r}w^{-1}):(u^rv^{3r}w^{-2})=u^{r-r}\cdot v^{-2r-3r}\cdot w^{-1-(-2)}=u^0v^{-5r}w^1=\boxed{\frac{w}{v^{5r}}}$

Velká čísla: $1\,000\,000=10^6$, $1\,000\,000\,000=10^9$ ... miliarda

10^{12} ... bilion (má 2krát více nul než milion), 10^{18} trilion (má 3krát více nul než milion).

 řecky: bi=2, tri=3.

Odmocniny s částečným zavedením racionálního exponentu

Je-li $a, b \in R$, $m, n \in N$, $k \in Z$ platí tyto věty:

Věta 1: Jsou-li $a_1, a_2, a_3, \ldots, a_s$ kladná čísla, pak

$$\boxed{\sqrt[n]{a_1 a_2 a_3 \cdots a_s} = \sqrt[n]{a_1} \cdot \sqrt[n]{a_2} \cdot \sqrt[n]{a_3} \cdots \sqrt[n]{a_s}}$$

například: $\sqrt{36 \cdot 25} = \sqrt{36} \cdot \sqrt{25} = 6 \cdot 5 = 30$

$$\sqrt[3]{8 \cdot 125 \cdot 27} = \sqrt[3]{8} \cdot \sqrt[3]{125} \cdot \sqrt[3]{27} = 2 \cdot 5 \cdot 3 = 30$$

Věta 2. Jestliže $a \in R^+$, $s \in N$, pak

$$\boxed{\sqrt[n]{a^s} = \left(\sqrt[n]{a}\right)^s = a^{\frac{s}{n}}}$$

$\sqrt{x}$ lze psát $\sqrt[2]{x}$

například: $\sqrt{4^3} = \left(\sqrt{4}\right)^3 = 2^3 = 8$, nebo

$$\left.\begin{aligned} &\sqrt{(2^2)^3} = \sqrt{2^6} = \sqrt{64} = 8, \text{ nebo} \\ &\sqrt{4^3} = \sqrt[2]{2^6} = 2^{\frac{6}{2}} = 2^3 = 8 \end{aligned}\right\} \text{tři možnosti výpočtu}$$

Věta 3: Jsou-li $a, b \in R^+$, pak

R^+ jsou kladná reálná čísla

$$\boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$$

, například:

$$\sqrt[2]{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4} \qquad \frac{\sqrt{63}}{\sqrt{7}} = \sqrt{\frac{63}{7}} = \sqrt{9} = 3$$

$$\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{4}{7} \qquad \frac{\sqrt[5]{a^3b^2}}{\sqrt[5]{a^2b^3}} = \sqrt[5]{\frac{a^3b^2}{a^2b^3}} = \sqrt[5]{\frac{a}{b}}, \ a>0, b>0$$

Věta 4: Je-li $a \in R^+$, $m, n \in N$, pak

$$\boxed{\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}}$$

ve vztahu k rac. exponentům:

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{\sqrt[n]{a^1}} = \sqrt[m]{a^{\frac{1}{n}}} = \left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{mn}} = \sqrt[mn]{a^1} = \sqrt[mn]{a}$$

Například:

$\sqrt[3]{\sqrt{5}} = \sqrt[3]{\sqrt[2]{5}} = \sqrt[6]{5}$ | $\sqrt{\sqrt[3]{25}} = \sqrt[3]{\sqrt{25}} = \sqrt[3]{5}$

$\sqrt[3]{\sqrt{x^3}} = \sqrt[3]{\sqrt[2]{x^3}} = \sqrt[2]{\sqrt[3]{x^3}} = \sqrt[2]{x} = \sqrt{x}$, $x>0$ | $\sqrt[3]{\sqrt[5]{2}} = \sqrt[15]{2}$

$\sqrt[4]{16} = \sqrt{\sqrt{16}} = \sqrt{4} = 2$ | $\sqrt[6]{64} = \sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$

$\sqrt[4]{121} = \sqrt[4]{11^2} = \sqrt[2]{\sqrt[2]{11^2}} = \sqrt[2]{11} = \sqrt{11}$

Věta 5: Je-li $a \in R^+$, $m, n, p \in N$, pak

Ⓐ $\boxed{\sqrt[np]{a^{mp}} = \sqrt[n]{a^m}}$

Ⓑ $\boxed{\sqrt[n]{a} \cdot \sqrt[m]{a} = \sqrt[mn]{a^{m+n}}}$, což lze zdůvodnit pomocí rac. exponentů

$$\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \cdot a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$$

Například:

$\sqrt[6]{a^9} = \sqrt[3\cdot 2]{a^{3\cdot 3}}$ = Podle Ⓐ lze „krátit" 3 proti 3 $= \sqrt[2]{a^3} = \sqrt{a^3}$

$\sqrt[4]{9} = \sqrt[4]{3^2} = \sqrt[2\cdot 2]{3^2} = \sqrt{3}$ | $\sqrt[6]{a^9 b^3} = \sqrt[6]{(a^3 b)^3} = \sqrt[2]{a^3 b} = \sqrt{a^3 b}$

$\sqrt[3]{a^{12}} = \sqrt[3]{a^{4\cdot 3}} = \sqrt[1]{a^4} = a^4$ | $\sqrt[12]{a^4} = \sqrt[4\cdot 3]{a^4} = \sqrt[3]{a^1} = \sqrt[3]{a}$

$\sqrt[18]{\frac{a^9}{b^{27}}} = \sqrt[2\cdot 9]{\left(\frac{a}{b^3}\right)^9} = \sqrt[2]{\frac{a}{b^3}} = \sqrt{\frac{a}{b^3}}$

Příklad 13: Vypočítejte:

a) $\sqrt{\sqrt[5]{9}} = \sqrt[5]{\sqrt{9}} = \sqrt[5]{3}$ | b) $\sqrt[6]{64} = \sqrt[3]{\sqrt[2]{64}} = \sqrt[3]{8} = 2$

c) $\sqrt[10]{4a^6 b^8} = \sqrt[2\cdot 5]{(2a^3 b^4)^2} = \sqrt[5]{2a^3 b^4}$ | d) $\frac{\sqrt{13}}{\sqrt{52}} = \sqrt{\frac{13}{52}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

e) $\sqrt{5} \cdot \sqrt[3]{5} \cdot \sqrt[4]{5} = \sqrt[12]{5^6} \cdot \sqrt[12]{5^4} \cdot \sqrt[12]{5^3} = \sqrt[12]{5^6 \cdot 5^4 \cdot 5^3} = \sqrt[12]{5^{13}}$ lze částečně odmocnit $\sqrt[12]{5^{12} \cdot 5} = \sqrt[12]{5^{12}} \cdot \sqrt[12]{5} = 5 \cdot \sqrt[12]{5}$ $n(2,3,4) = 12$

f) $5\sqrt{3}+9\sqrt{3}-4\sqrt{3}=\sqrt{3}(5+9-4)=10\cdot\sqrt{3}$

g) $\sqrt{2}:\sqrt[3]{2} = \sqrt[2]{2^1}:\sqrt[3]{2^1}$ ... upravíme na spol. násobek 2, 3, tj. 6

$= \sqrt[2\cdot3]{2^{1\cdot3}} : \sqrt[3\cdot2]{2^{1\cdot2}} = \sqrt[6]{2^3}:\sqrt[6]{2^2} = \sqrt[6]{\frac{2^3}{2^2}} = \sqrt[6]{2}$

h) $\sqrt[3]{10\,000}\cdot\sqrt[3]{1,4}\cdot\sqrt[3]{0,196} = \sqrt[3]{10^4}\cdot\sqrt[3]{14\cdot10^{-1}}\cdot\sqrt[3]{\underbrace{196}_{14^2}\cdot10^{-3}} =$

$=\sqrt[3]{10^{4-1-3}\cdot14\cdot14^2} = \sqrt[3]{10^0\cdot14^3} = \sqrt[3]{14^3} = 14$

i) $\sqrt[3]{\frac{27}{1000}} = \sqrt[3]{\frac{3^3}{10^3}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{10^3}} = \frac{3}{10}$

j) $\sqrt[4]{0,0016} = \sqrt[4]{16\cdot10^{-4}} = \sqrt[4]{16}\cdot\underbrace{\sqrt[4]{10^{-4}}}_{} = \sqrt[4]{2^4}\cdot10^{-1} = 2\cdot10^{-1} = 0,2$

Pomocí lomeného exponentu

$10^{-\frac{4}{4}} = 10^{-1}$

k) $\sqrt[3]{7+\sqrt{22}}\cdot\sqrt[3]{7-\sqrt{22}} = \sqrt[3]{\underbrace{(7+\sqrt{22})\cdot(7-\sqrt{22})}_{(A+B)(A-B)=A^2-B^2}} = \sqrt{49-22} = \sqrt[3]{27} = 3$

l) $\sqrt{\sqrt[3]{\sqrt{c^{10}}}} = \sqrt[2]{\sqrt[3]{\sqrt[2]{c^{10}}}} = \sqrt[12]{c^{10}} \quad \sqrt[2\cdot6]{c^{2\cdot5}} = \sqrt[6]{c^5}$

Následuje výběr několika úloh ze sbírky pro OA

Příklad 14: Vypočítejte:

a) 3a/130A: $3\sqrt{5}-4\sqrt{5}+3(\sqrt{5}-1)=3\sqrt{5}-4\sqrt{5}+3\sqrt{5}-3=\boxed{2\sqrt{5}-3}$

b) 3d/130A: $3\sqrt{2}-2\sqrt[3]{2}+3(\sqrt{2}-\sqrt[3]{2})=3\sqrt{2}-2\sqrt[3]{2}+3\sqrt{2}-3\sqrt[3]{2}=\boxed{6\sqrt{2}-5\sqrt[3]{2}}$

c) 3e/130A: $\frac{\sqrt{2}-\sqrt{3}}{4}-\frac{\sqrt{3}+\sqrt{2}}{5}-2(\sqrt{3}+2\sqrt{2})=$

$=\frac{1}{4}\sqrt{2}-\frac{1}{4}\sqrt{3}-\frac{1}{5}\sqrt{3}-\frac{1}{5}\sqrt{2}-2\sqrt{3}+2\sqrt{2}=\sqrt{2}\left(\frac{1}{4}-\frac{1}{5}-4\right)+\sqrt{3}\left(-\frac{1}{4}-\frac{1}{5}-2\right)=$

$=\boxed{-\frac{79}{20}\sqrt{2}-\frac{49}{20}\sqrt{3} = \frac{-79\sqrt{2}-49\sqrt{3}}{20}}$ dva rovnocenné výsledky

Příklad 15: Vynásobte a zjednodušte.

a) 4a/130A: $(\sqrt{18}-\sqrt{32}+\sqrt{8}+\sqrt{50})\cdot\sqrt{2}=\sqrt{36}-\sqrt{64}+\sqrt{16}+\sqrt{100}=$

$=6-8+4+10=\boxed{12}$

---

b) 5a/130A: $(\sqrt{8}+\sqrt{3})\cdot(\sqrt{8}-\sqrt{3})=(\sqrt{8})^2-(\sqrt{3})^2=\sqrt{8^2}-\sqrt{3^2}=$

$(A+B)(A-B)=A^2-B^2$ $\qquad =8-3=\boxed{5}$

---

c) 5c/130A: $(5-\sqrt{15})\cdot(5+\sqrt{15})=5^2-(\sqrt{15})^2=25-15=\boxed{10}$

---

d) 5d/130A: $(4-2\sqrt{2})\cdot(4+2\sqrt{2})=4^2-\underbrace{(2\sqrt{2})^2}_{2^2\cdot(\sqrt{2})^2}=16-4\cdot2=16-8=\boxed{8}$

---

e) 6a/130A:

$$\frac{(2+3\sqrt{3})(2-3\sqrt{3})}{(\sqrt{3}+2)\cdot(\sqrt{3}-2)}=\frac{4-9\cdot3}{3-4}=\frac{4-27}{-1}=\frac{-23}{-1}=\boxed{23}$$

---

f) 6d/130A:

$$\frac{7+\sqrt{3}}{6+\sqrt{3}}-\frac{7-\sqrt{3}}{6-\sqrt{3}}=\frac{(7+\sqrt{3})\cdot(6-\sqrt{3})-(7-\sqrt{3})\cdot(6+\sqrt{3})}{(6+\sqrt{3})\cdot(6-\sqrt{3})}=$$

$$=\frac{42+6\sqrt{3}-7\sqrt{3}-\sqrt{3}\cdot\sqrt{3}-(42-6\sqrt{3}+7\sqrt{3}-\sqrt{3}\cdot\sqrt{3})}{36-3}=\frac{42-\sqrt{3}-3-(42+\sqrt{3}-3)}{33}=$$

$$=\frac{\cancel{42}-\sqrt{3}-3-\cancel{42}-\sqrt{3}+3}{33}=\boxed{\frac{-2\sqrt{3}}{33}}$$

Příklad 16: (7/130A) Částečně odmocněte: (Též 8/14)

a) $\sqrt{75}=\sqrt{3\cdot25}=\sqrt{3}\cdot\sqrt{25}=\boxed{5\cdot\sqrt{3}}$ | $\sqrt{32}=\sqrt{2\cdot16}=\boxed{4\cdot\sqrt{2}}$

$\sqrt{300}=\sqrt{100\cdot3}=\boxed{10\cdot\sqrt{3}}$ | $\sqrt{48}=\sqrt{3\cdot16}=\boxed{4\sqrt{3}}$

b) $\sqrt{4a+4b}=\sqrt{4(a+b)}=\boxed{2\sqrt{a+b}}$ | $\sqrt{64x-64}=\sqrt{64(x-1)}=\boxed{8\sqrt{x-1}}$

$\sqrt{81x-81y}=\sqrt{81(x-y)}=\boxed{9\sqrt{x-y}}$

c) 8d/140A: $3\sqrt{45}-2\cdot\sqrt{180}+3\sqrt{80}-\sqrt{20}=3\cdot\sqrt{9\cdot5}-2\cdot\sqrt{5\cdot36}+3\sqrt{5\cdot16}-\sqrt{4\cdot5}=$

$=3\cdot3\sqrt{5}-2\cdot6\sqrt{5}+3\cdot4\sqrt{5}-2\sqrt{5}=$ $\quad 9\sqrt{5}-\cancel{12\sqrt{5}}+\cancel{12\sqrt{5}}-2\sqrt{5}=\boxed{7\sqrt{5}}$

d) není OA: $\sqrt[3]{135} = \sqrt[3]{5\cdot 27} = \sqrt[3]{5}\cdot\sqrt[3]{27} = \boxed{3\cdot\sqrt[3]{5}}$

Příklad 17 (9/14 OA):

$$\overset{(A+B)^2}{\uparrow}$$

a) $$(\underbrace{\sqrt{2}+\sqrt{3}}_{(A} + \underbrace{\sqrt{5}}_{+B)})\cdot(\underbrace{\sqrt{2}+\sqrt{3}}_{(A} - \underbrace{\sqrt{5}}_{-B)}) = (\sqrt{2}+\sqrt{3})^2 - (\sqrt{5})^2 =$$

$$= \left[(\sqrt{2})^2 + 2\sqrt{2}\cdot\sqrt{3} + (\sqrt{3})^2\right] - (\sqrt{5})^2 = (2+2\sqrt{6}+3) - 5 = 2+3-5+2\sqrt{6} = \boxed{2\sqrt{6}}$$

b) $$(\underbrace{2\sqrt{5}-3\sqrt{3}}_{(A} + \underbrace{\sqrt{30}}_{+B)})\cdot(\underbrace{2\sqrt{5}-3\sqrt{3}}_{(A} - \underbrace{\sqrt{30}}_{-B)}) = (2\sqrt{5}-3\sqrt{3})^2 - (\sqrt{30})^2 =$$

$$= (2\sqrt{5})^2 - 2\cdot 12\sqrt{5}\cdot\sqrt{3} + (3\sqrt{3})^2 - (\sqrt{30})^2 = 4\cdot 5 - 12\sqrt{15} + 9\cdot 3 - 30 =$$

$$= 20 - 12\sqrt{15} + 27 - 30 = \boxed{17 - 12\sqrt{15}}$$

Příklad 18: Usměrněte zlomky (což znamená odstranit odmocniny ze jmenovatelů zlomků) a zjednodušte:

a) 10a/14OA: $$\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{\sqrt{15}}{5}$$

b) 10d/14OA: $$\frac{2+\sqrt{6}}{2\cdot\sqrt{3}} = \frac{(2+\sqrt{6})\cdot\sqrt{3}}{2\cdot\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}+\sqrt{18}}{2(\sqrt{3})^2} = \frac{2\sqrt{3}+\sqrt{18}}{2\cdot 3} = \boxed{\frac{2\sqrt{3}+3\sqrt{2}}{6}}$$

c) 10f/14OA: $$\frac{3}{2-\sqrt{3}} = \frac{3\,(2+\sqrt{3})}{(2-\sqrt{3})\cdot(2+\sqrt{3})} = \frac{3(2+\sqrt{3})}{4-3} = \boxed{3(2+\sqrt{3})}$$

d) 11b/14OA: $$\frac{3\sqrt{10}}{4\sqrt{5}-5\cdot\sqrt{2}} = \frac{3\sqrt{10}\cdot(4\sqrt{5}+5\sqrt{2})}{(4\sqrt{5}-5\sqrt{2})\cdot(4\sqrt{5}+5\sqrt{2})} = \frac{12\sqrt{50}+15\sqrt{20}}{(4\sqrt{5})^2-(5\sqrt{2})^2} =$$

$$= \frac{12\cdot\sqrt{2\cdot 25}+15\sqrt{4\cdot 5}}{16\cdot 5 - 25\cdot 2} = \frac{12\cdot 5\sqrt{2}+15\cdot 2\sqrt{5}}{30} = \frac{60\sqrt{2}+30\sqrt{5}}{30} = \frac{30(2\sqrt{2}+\sqrt{5})}{30} =$$

$$= \boxed{2\sqrt{2}+\sqrt{5}}$$

e) 12a/14OA $$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})^2+(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}+\sqrt{3})\cdot(\sqrt{5}-\sqrt{3})} =$$

$$= \frac{(5-\cancel{2\sqrt{15}}+3)+(5+\cancel{2\sqrt{15}}+3)}{5-3} = \frac{8+8}{2} = \frac{16}{2} = \boxed{8}$$

Příklad 19 (z jiných zdrojů): Usměrněte zlomky.

a) $$\frac{1}{\sqrt[3]{2}} = \frac{1\cdot\sqrt[3]{2^2}}{\sqrt[3]{2}\cdot\sqrt[3]{2^2}} = \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \boxed{\frac{\sqrt[3]{4}}{2}}$$

b) $$\frac{2}{\sqrt[3]{9}} = \frac{2}{\sqrt[3]{3^2}}\cdot\frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{3^3}} = \boxed{\frac{2\cdot\sqrt[3]{3}}{3}}$$

c) $$\frac{1}{\sqrt[3]{50}} = \frac{1}{\sqrt[3]{2\cdot5^2}}\cdot\frac{1\cdot\sqrt[3]{2^2\cdot5}}{\sqrt[3]{2^2\cdot5}} = \frac{\sqrt[3]{20}}{\sqrt[3]{2^3\cdot5^3}} = \frac{\sqrt[3]{20}}{\sqrt[3]{2^3}\cdot\sqrt[3]{5^3}} = \frac{\sqrt[3]{20}}{2\cdot5} = \boxed{\frac{\sqrt[3]{20}}{10}}$$

d) $$\frac{12}{\sqrt[3]{18}} = \frac{12}{\sqrt[3]{2\cdot3^2}} = \frac{12}{\sqrt[3]{2}\cdot\sqrt[3]{3^2}} = \frac{12\cdot\sqrt[3]{2^2}\cdot\sqrt[3]{3}}{\sqrt[3]{2}\cdot\sqrt[3]{3^2}\cdot\sqrt[3]{2^2}\cdot\sqrt[3]{3}} = \frac{12\sqrt[3]{12}}{\sqrt[3]{2^3}\cdot\sqrt[3]{3^3}} = \frac{12\cdot\sqrt[3]{12}}{2\cdot3} = \boxed{2\cdot\sqrt[3]{12}}$$

Mocniny s racionálními exponenty (mocniteli)

Definice: Pro $a>0$, $m\in Z$, $n\in N$ je

$$\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

Například: $\sqrt[3]{x^2} = x^{\frac{2}{3}}$, $\sqrt{a} = \sqrt[2]{a^1} = a^{\frac{1}{2}}$

$$\sqrt{2}:\sqrt[3]{2} = 2^{\frac{1}{2}}:2^{\frac{1}{3}} = 2^{\frac{1}{2}-\frac{1}{3}} = 2^{\frac{1}{6}} = \boxed{\sqrt[6]{2}}$$

$$\sqrt{a\sqrt[3]{b}}\cdot\sqrt[3]{b\sqrt{a}} = (ab^{\frac{1}{3}})^{\frac{1}{2}}\cdot(ba^{\frac{1}{2}})^{\frac{1}{3}} = a^{\frac{1}{2}}b^{\frac{1}{6}}\cdot b^{\frac{1}{3}}\cdot a^{\frac{1}{6}} =$$
$$a^{\frac{1}{2}+\frac{1}{6}}\cdot b^{\frac{1}{3}+\frac{1}{6}} = a^{\frac{2}{3}}\cdot b^{\frac{1}{2}} = \boxed{\sqrt[3]{a^2}\cdot\sqrt{b}},\quad a>0, b>0$$

$$\sqrt[3]{\frac{x}{y}\sqrt[4]{\frac{y}{x}}} = \left[\frac{x}{y}\cdot\left(\frac{y}{x}\right)^{\frac{1}{4}}\right]^{\frac{1}{3}} = \left[\frac{x^1}{y^1}\cdot\left(\frac{x}{y}\right)^{-\frac{1}{4}}\right]^{\frac{1}{3}} = \left[\frac{x^{1-\frac{1}{4}}}{y^{1-\frac{1}{4}}}\right]^{\frac{1}{3}} = \left[\frac{x^{\frac{3}{4}}}{y^{\frac{3}{4}}}\right]^{\frac{1}{3}} =$$
$$\left[\left(\frac{x}{y}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}} = \left(\frac{x}{y}\right)^{\frac{1}{4}} = \boxed{\sqrt[4]{\frac{x}{y}}}$$

Příklad 20: Vypočítejte:

$27^{\frac{1}{3}} = \sqrt[3]{27} = \boxed{3}$ | $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = \boxed{4}$ | $9^{-\frac{1}{2}} = \sqrt{9^{-1}} = \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$

$$125^{-\frac{2}{3}} = \sqrt[3]{125^{-2}} = \sqrt[3]{\frac{1}{125^2}} = \frac{\sqrt[3]{1}}{\sqrt[3]{125^2}} = \frac{1}{\sqrt[3]{(5^3)^2}} = \frac{1}{\sqrt[3]{5^6}} = \frac{1}{5^{\frac{6}{3}}} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$$

Příklad 21: Vyjádřete výrazem s jednou odmocninou

NÁROČNÝ

$$\sqrt{x\cdot\sqrt[3]{x^6\cdot\sqrt[4]{x^3}}} : \sqrt[5]{x^4\cdot\sqrt[4]{x}\cdot\sqrt[6]{x^7}}$$, kde $x>0$

$$\left[x\left(x^6\cdot x^{\frac{3}{4}}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}} : \left(x^4\cdot x^{\frac{1}{4}}\cdot x^{\frac{7}{6}}\right) = x^{\frac{1}{2}}\cdot\left(x^{\frac{27}{4}}\right)^{\frac{1}{6}} : \left(x^{\frac{65}{12}}\right)^{\frac{1}{5}} =$$

$$= \left(x^{\frac{1}{2}}\cdot x^{\frac{27}{24}}\right) : \left(x^{\frac{65}{60}}\right) = x^{\frac{1}{2}+\frac{27}{4}} : x^{\frac{13}{12}} = x^{\frac{13}{8}} : x^{\frac{13}{12}} = x^{\frac{13}{8}-\frac{13}{12}} = x^{\frac{13}{24}} = \boxed{\sqrt[24]{x^{13}}}$$

Příklad 22 = výběr úloh ze sbírky pro OA na str. 15 a 16

a) 1a/150A: $x^{-\frac{1}{2}}\cdot x^{\frac{3}{4}} = x^{-\frac{1}{2}+\frac{3}{4}} = x^{\frac{1}{4}} = \boxed{\sqrt[4]{x}}$

b) 1b/150A: $x^{\frac{3}{2}}\cdot x^{-1} = x^{\frac{3}{2}-1} = x^{\frac{1}{2}} = \boxed{\sqrt{x}}$

c) 1f/150A:

$$\frac{\sqrt{x}\cdot\sqrt[3]{y}}{\sqrt{xy}} = \frac{x^{\frac{1}{2}}\cdot y^{\frac{1}{3}}}{(xy)^{\frac{1}{2}}} = x^{\frac{1}{2}}\cdot y^{\frac{1}{3}}\cdot(xy)^{-\frac{1}{2}} = x^{\frac{1}{2}}\cdot y^{\frac{1}{3}}\cdot x^{-\frac{1}{2}}\cdot y^{-\frac{1}{2}} =$$

$$= x^{\frac{1}{2}-\frac{1}{2}}\cdot y^{\frac{4}{3}-\frac{1}{2}} = x^0\cdot y^{-\frac{1}{6}} = \boxed{y^{-\frac{1}{6}}}$$ nebo $\frac{1}{y^{\frac{1}{6}}} = \boxed{\frac{1}{\sqrt[6]{y}}}$ ale ještě usměrnit / lekce

$$\frac{1\cdot\sqrt[6]{y^5}}{\sqrt[6]{y}\cdot\sqrt[6]{y^5}} = \frac{\sqrt[6]{y^5}}{\sqrt[6]{y^6}} = \boxed{\frac{\sqrt[6]{y^5}}{y}}$$

d) 2b/150A: $0{,}0001^{-\frac{3}{4}} = \left(\frac{1}{10\,000}\right)^{-\frac{3}{4}} = (10\,000)^{\frac{3}{4}} = \left(10^4\right)^{\frac{3}{4}} = 10^{\frac{12}{4}} = \boxed{10^3}$

e) 2c/150A: $25^{0,5} = 25^{\frac{1}{2}} = \sqrt{25} = \boxed{5}$

f) 3a/150A: $\sqrt{2}\cdot\sqrt[3]{2} = 2^{\frac{1}{2}}\cdot 2^{\frac{1}{3}} = 2^{\frac{1}{2}+\frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \boxed{\sqrt[6]{32}}$

g) 3e/150A: $\sqrt{2\sqrt{2\cdot\sqrt{2}}}$ = Postupně platí: $\sqrt[2]{2\cdot\sqrt[2]{2\cdot 2^{\frac{1}{2}}}} = \sqrt[2]{2\cdot\left(2\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}}} =$

$$= \left[2\cdot\left(2\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left(2\cdot 2^{\frac{1}{2}}\cdot 2^{\frac{1}{4}}\right)^{\frac{1}{2}} = \left(2^{1+\frac{1}{2}+\frac{1}{4}}\right)^{\frac{1}{2}} = \left(2^{\frac{7}{4}}\right)^{\frac{1}{2}} = 2^{\frac{7}{8}} = \sqrt[8]{2^7} = \boxed{\sqrt[8]{128}}$$

h) 3f/150A: $(\sqrt{2}\cdot\sqrt[3]{2})^{-\frac12} = (2^{\frac12}\cdot 2^{\frac13})^{-\frac12}$ = umocníme jako součin

$$= 2^{\frac12\cdot(-\frac12)}\cdot 2^{\frac13\cdot(-\frac12)} = 2^{-\frac14}\cdot 2^{-\frac16} = 2^{-\frac14-\frac16} = 2^{-\frac{5}{12}} = \frac{1}{2^{\frac{5}{12}}} = \frac{1}{\sqrt[12]{2^5}} =$$

$$= \boxed{\frac{1}{\sqrt[12]{32}}}$$

---

i) 4a/150A: $$\frac{12^{-\frac12}\cdot 6^{\frac13}}{3^{-1}} = (2.2.3)^{-\frac12}\cdot(2.3)^{\frac13}\cdot 3^1 = 2^{-\frac12}\cdot 2^{-\frac12}\cdot 3^{-\frac12}\cdot 2^{\frac13}\cdot 3^{\frac13}\cdot 3^1 =$$

$$= 2^{-\frac12-\frac12+\frac13}\cdot 3^{-\frac12+\frac13+1} = \boxed{2^{-\frac23}\cdot 3^{\frac56}}$$

---

j) 4b/150A: $$\frac{15^{-\frac13}\cdot 25^{-2}}{\sqrt{9}} = \frac{(3.5)^{-\frac13}\cdot(5.5)^{-2}}{3} = 3^{-\frac13}\cdot 5^{-\frac13}\cdot 5^{-2}\cdot 5^{-2}\cdot 3^{-1} =$$

$$= 3^{-\frac13-1}\cdot 5^{-\frac13-2-2} = \boxed{3^{-\frac43}\cdot 5^{-\frac{13}{3}}}$$

---

k) 4c/150A: $$\frac{7^{-\frac13}\cdot 49^{\frac12}}{\sqrt[3]{14}} = \frac{7^{-\frac13}\cdot(7.7)^{\frac12}}{(14)^{\frac13}} = 7^{-\frac13}\cdot 7^{\frac12}\cdot 7^{\frac12}\cdot(2.7)^{-\frac13} =$$

$$= 7^{-\frac13}\cdot 7^{\frac12}\cdot 7^{\frac12}\cdot 2^{-\frac13}\cdot 7^{-\frac13} = 2^{-\frac13}\cdot 7^{-\frac13+\frac12+\frac12-\frac13} = 2^{-\frac13}\cdot 7^{\frac13} = \frac{7^{\frac13}}{2^{\frac13}} = \left(\frac72\right)^{\frac13} = \boxed{\sqrt[3]{\frac72}}$$

---

l) 5a/150A: $$\frac{(x^{\frac12}\cdot y^{\frac23})^{-3}}{(x^3\cdot y^{-1})^{-\frac12}} = \frac{\cancel{x^{-\frac32}}\cdot y^{-2}}{\cancel{x^{-\frac32}}\cdot y^{\frac12}} = y^{-2}\cdot y^{-\frac12} = y^{-\frac52} = \frac{1}{y^{\frac52}} = \frac{1}{\sqrt{y^5}} =$$

$= \frac{1}{\sqrt{y^4\cdot y}} = \frac{1}{\sqrt{y^4}\cdot\sqrt{y}} = \frac{1}{y^{\frac42}\cdot\sqrt{y}} = \frac{1}{y^2\cdot\sqrt{y}}$ … usměrníme $= \frac{1\sqrt{y}}{y^2\cdot\sqrt{y}\cdot\sqrt{y}} =$

$$= \frac{\sqrt{y}}{y^2\cdot y} = \boxed{\frac{\sqrt{y}}{y^3}}$$

---

m) 6b/150A: $$\frac{x^{-\frac34}\cdot y^{\frac57}\cdot\sqrt{x^2 y^{\frac12}}}{xy\cdot\sqrt{xy}} = \frac{x^{-\frac34}\cdot y^{\frac57}\cdot(x^2 y^{\frac12})^{\frac12}}{xy\cdot(xy)^{\frac12}} = \frac{x^{-\frac34}\cdot y^{\frac57}\cdot x^1\cdot y^{\frac14}}{x^1\cdot y^1\cdot x^{\frac12}\cdot y^{\frac12}} =$$

$$= \frac{x^{\frac14}\cdot y^{\frac{27}{28}}}{x^{\frac32}\cdot y^{\frac32}} = x^{\frac14}\cdot y^{\frac{27}{28}}\cdot x^{-\frac32}\cdot y^{-\frac32} = \boxed{x^{-\frac54}\cdot y^{-\frac{15}{28}}}$$

n) 9116 OA: $\sqrt{a\sqrt[3]{b^{-1}}} : \sqrt[3]{b^2\sqrt{a}} + \sqrt[6]{b} : b =$

$$= \frac{(a\cdot b^{-\frac{1}{3}})^{\frac{1}{2}}}{(b^2 a^{\frac{1}{2}})^{\frac{1}{3}}} + b^{\frac{1}{6}} : b = \frac{a^{\frac{1}{2}}\cdot b^{-\frac{1}{6}}}{a^{\frac{1}{6}}\cdot b^{\frac{2}{3}}} + b^{\frac{1}{6}-1} =$$

$$= a^{\frac{1}{2}}\cdot b^{-\frac{1}{6}}\cdot a^{-\frac{1}{6}}\cdot b^{-\frac{2}{3}} + b^{-\frac{5}{6}} = a^{\frac{1}{3}}\cdot b^{-\frac{5}{6}} + b^{-\frac{5}{6}} = b^{-\frac{5}{6}}(a^{\frac{1}{3}}+1)$$

$$= \boxed{\frac{\sqrt[3]{a}+1}{\sqrt[6]{b^5}}}$$

Příklad 23 (z jiných zdrojů než OA)

Zjednodušte na výraz s jednou odmocninou:

a) $$\sqrt[3]{\frac{a^2}{\sqrt{b^3}}}\cdot\sqrt{\frac{b}{\sqrt[3]{a}}} = \left(\frac{a^2}{b^{\frac{3}{2}}}\right)^{\frac{1}{3}}\cdot\left(\frac{b}{a^{\frac{1}{3}}}\right)^{\frac{1}{2}} = \frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\cdot\frac{b^{\frac{1}{2}}}{a^{\frac{1}{6}}} =$$

$$= a^{\frac{2}{3}}\cdot b^{-\frac{1}{2}}\cdot b^{\frac{1}{2}}\cdot a^{-\frac{1}{6}} = a^{\frac{1}{3}}\cdot b^{0} = \boxed{\sqrt[3]{a}}$$

---

b) $$\sqrt{x^2\sqrt[3]{y}} : \sqrt[3]{\frac{x\sqrt{x}}{y}} = (x^2\cdot y^{\frac{1}{3}})^{\frac{1}{2}} : \left(\frac{x^1\cdot x^{\frac{1}{2}}}{y}\right)^{\frac{1}{3}} =$$

$$= x\cdot y^{\frac{1}{6}} : \left(\frac{x^{\frac{3}{2}}}{y}\right)^{\frac{1}{3}} = xy^{\frac{1}{6}} : \frac{x^{\frac{1}{2}}}{y^{\frac{1}{3}}} = xy^{\frac{1}{6}}\cdot\frac{y^{\frac{1}{3}}}{x^{\frac{1}{2}}} =$$

$$= x\cdot x^{-\frac{1}{2}}\cdot y^{\frac{1}{6}}\cdot y^{\frac{1}{3}} = x^{\frac{1}{2}}\cdot y^{\frac{1}{2}} = (xy)^{\frac{1}{2}} = \boxed{\sqrt{xy}}$$

Příklad 24: Vyjádřete jako mocniny:

$$\sqrt{\frac{a\cdot\sqrt[3]{b}}{\sqrt[3]{a\sqrt{b}}}} = \left[\frac{a\cdot b^{\frac{1}{3}}}{(a\cdot b^{\frac{1}{2}})^{\frac{1}{3}}}\right]^{\frac{1}{2}} = \frac{(a\cdot b^{\frac{1}{3}})^{\frac{1}{2}}}{(a^{\frac{1}{3}}\cdot b^{\frac{1}{6}})^{\frac{1}{2}}} = \frac{a^{\frac{1}{2}}\cdot b^{\frac{1}{6}}}{a^{\frac{1}{6}}\cdot b^{\frac{1}{12}}} =$$

$$= a^{\frac{1}{2}}\cdot a^{-\frac{1}{6}}\cdot b^{\frac{1}{6}}\cdot b^{-\frac{1}{12}} = \boxed{a^{\frac{1}{3}} b^{\frac{1}{12}}}$$