19a) $\boxed{\text{BINOMICKÁ ROVNICE}}$

Binomická rovnice má tvar

I $\boxed{x^n - a = 0}$, respektive $\boxed{x^n - |a|(\cos\alpha + i\sin\alpha) = 0}$ II.

kde $a \in C$ (množiny komplexních čísel), $n \in N - \{1\}$

$x^3 - i = 0$ $(x^3 = i)$, $x^3 + 27 = 0$, $x^5 - 1 - i\sqrt{3} = 0$ aj.

Rovnice $x^n - |a|(\cos\alpha + i\sin\alpha) = 0$ má v oboru C n různých kořenů, a to

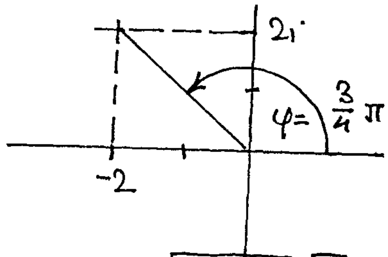
$$x_k = \sqrt[n]{|a|}\left(\cos\frac{\alpha + 2k\pi}{n} + i\sin\frac{\alpha + 2k\pi}{n}\right), \text{ kde } k = 0,1,2,3,\dots,(n-1)$$

Příklad 1: V množině C řešte rovnici $x^4 + 2 - 2i = 0$. (4/78-učeb.)

Rovnici upravíme na tvar I:

$x^4 - (-2+2i) = 0$ a kompl. číslo vyjádříme v goniometrickém tvaru

$$x^4 - \left[\sqrt{8}\left(\cos\tfrac{3}{4}\pi + i\sin\tfrac{3}{4}\pi\right)\right] = 0$$



$|-2+2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$-2+2i = \sqrt{8}\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right)$

$$x_k = \sqrt[4]{\sqrt[2]{8}}\left(\cos\frac{\frac{3}{4}\pi + 2k\pi}{4} + i\sin\frac{\frac{3}{4}\pi + 2k\pi}{4}\right); \; k = 0,1,2,3$$

$$x_k = (8^{\frac{1}{2}})^{\frac{1}{4}} \cdot \left[\cos\frac{\frac{3}{4}\pi}{\frac{4}{1}} + \frac{2}{4}k\pi + i\sin\frac{\frac{3}{4}\pi}{\frac{4}{1}} + \frac{2}{4}k\pi\right]$$

$$x_k = 8^{\frac{1}{8}}\left[\cos\left(\frac{3}{16}\pi + \frac{1}{2}k\pi\right) + i\sin\left(\frac{3}{16}\pi + \frac{1}{2}k\pi\right)\right]$$

$$x_k = \sqrt[8]{8}\left[\cos\left(\frac{3}{16}\pi + \frac{1}{2}\pi \cdot k\right) + i\sin\left(\frac{3}{16}\pi + \frac{1}{2}\pi \cdot k\right)\right]$$

dosadit postupně $k = 0,1,2,3$

Pro $k=0$ platí (atd.)

$$\boxed{x_0 = \sqrt[8]{8}\left(\cos\frac{3}{16}\pi + i\sin\frac{3}{16}\pi\right)}$$

$$x_1 = \sqrt[8]{8}\left[\cos\left(\frac{3}{16}\pi + \frac{1}{2}\pi \cdot 1\right) + i\sin\left(\frac{3}{16}\pi + \frac{1}{2}\pi \cdot 1\right)\right]$$

$$\boxed{x_1 = \sqrt[8]{8}\left(\cos\frac{11}{16}\pi + i\sin\frac{11}{16}\pi\right)}$$

$$x_2 = \sqrt[8]{8}\left[\cos\left(\frac{3}{16}\pi + \frac{1}{2}\pi\cdot 2\right) + i\sin\left(\frac{3}{16}\pi + \frac{1}{2}\pi\cdot 2\right)\right]$$

$$\boxed{x_2 = \sqrt[8]{8}\left(\cos\frac{19}{16}\pi + i\sin\frac{19}{16}\pi\right)}$$

$$x_3 = \sqrt[8]{8}\left[\cos\left(\frac{3}{16}\pi + \frac{1}{2}\pi\cdot 3\right) + i\sin\left(\frac{3}{16}\pi + \frac{1}{2}\pi\cdot 3\right)\right]$$

$$\boxed{x_3 = \sqrt[8]{8}\left(\cos\frac{27}{16}\pi + i\sin\frac{27}{16}\pi\right)}$$

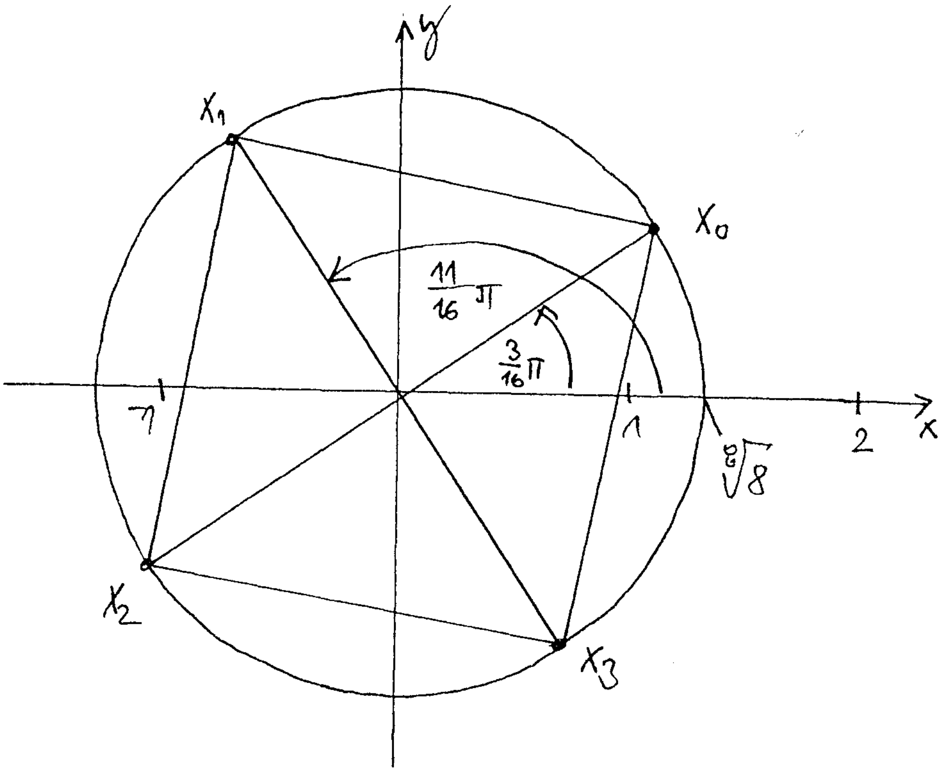
ČTI DŮLEŽITOU POZNÁMKU NA S. 14.

Grafické vyjádření v Gaussově rovině:

$\frac{3}{16}\pi = 33\frac{3}{4}°$ | $\frac{11}{16}\pi = 123\frac{3}{4}°$ | $\frac{19}{16}\pi = 213\frac{3}{4}°$ | $\frac{27}{16}\pi = 303\frac{3}{4}°$

$\sqrt[8]{8} \doteq 1{,}3$

Obrazy všech 4 kořenů dané binomické rovnice jsou vrcholy čtverce, které leží na kružnici s poloměrem $\sqrt[8]{8} \doteq 1{,}3$.

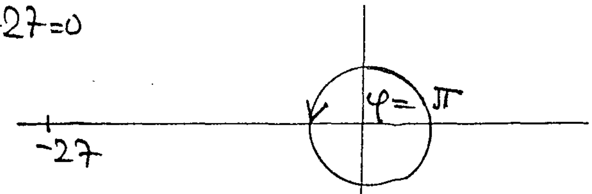


Příklad 2: (5/80 uč.) Řešte v C: $x^3 + 27 = 0$

$x^3 + 27 = 0$

$x^3 - (-27) = 0$

$x^3 - 27(\cos\pi + i\sin\pi) = 0$



$|-27| = \sqrt{27^2} = 27$

$x_k = \underbrace{\sqrt[3]{27}}_{3} \left(\cos \frac{\pi+2k\pi}{3} + i \sin \frac{\pi+2k\pi}{3}\right)$, kde $k=0,1,2$

$\boxed{x_0 = 3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)}$ nebo $\boxed{3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)}$ OPĚT MŮŽEŠ POUŽÍT POZNÁMKU na str. 14.

$x_1 = 3\left[\cos \frac{\pi+2\pi}{3} + i \sin \frac{\pi+2\pi}{3}\right]$

$x_1 = 3\left(\cos \frac{3\pi}{3} + i \sin \frac{3\pi}{3}\right) = \boxed{3(\cos\pi + i\sin\pi)} = 3(-1+0) = \boxed{-3}$

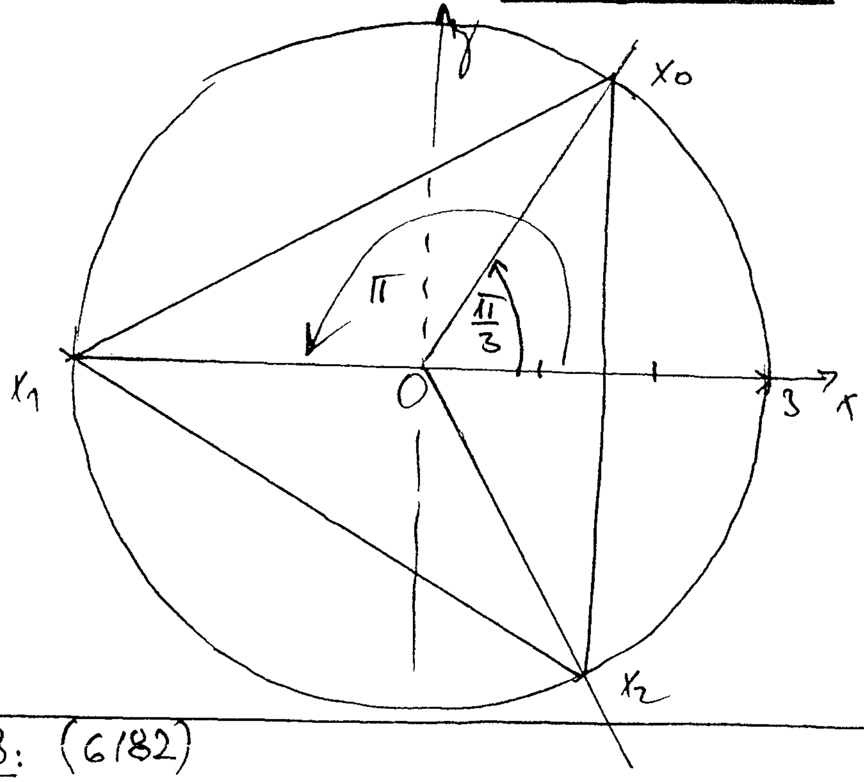
$x_2 = 3\left(\cos \frac{\pi+2\cdot 2\pi}{3} + i \sin \frac{\pi+2\cdot 2\pi}{3}\right)$

$x_2 = \boxed{3\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)} = \boxed{3\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)}$

$\frac{1}{3}\pi = 60°$

$\pi = 180°$

$\frac{5}{3}\pi = 300°$



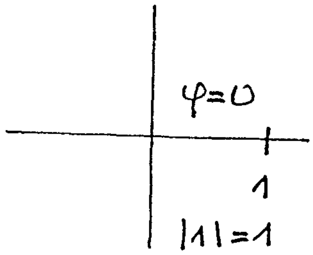
Obraz všech kořenů rovnice jsou vrcholy rovnostranného △ leží na kružnici $k(O;3)$.

Příklad 3: (6/82)

$x^6 - 1 = 0$

1.2 způsob:

$x^6 - (+1) = 0$



$x^6 - (\cos 0 + i \sin 0) \;\ldots\; x_k = \sqrt[6]{1}\cdot\left(\cos\frac{0+2k\pi}{6} + i \sin\frac{0+2k\pi}{6}\right)$

$x_k = \left(\cos \frac{1}{3}\pi\cdot k + i \sin \frac{1}{3}\pi k\right)$, kde $k=0,1,2,3,4,5$

$x_0 = (\cos 0 + i \sin 0) = 1+0 = 1$

$\boxed{x_0 = 1}$ ③

$x_1 = \cos\frac{1}{3}\pi\cdot 1 + i\sin\frac{1}{3}\pi\cdot 1 \quad \ldots \quad \boxed{x_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}} \quad \ldots \quad \frac{1}{3}\pi = 30^\circ$

$x_2 = \cos\frac{1}{3}\pi\cdot 2 + i\sin\frac{1}{3}\pi\cdot 2$

$x_2 = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi \quad \ldots \quad \boxed{x_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}}$

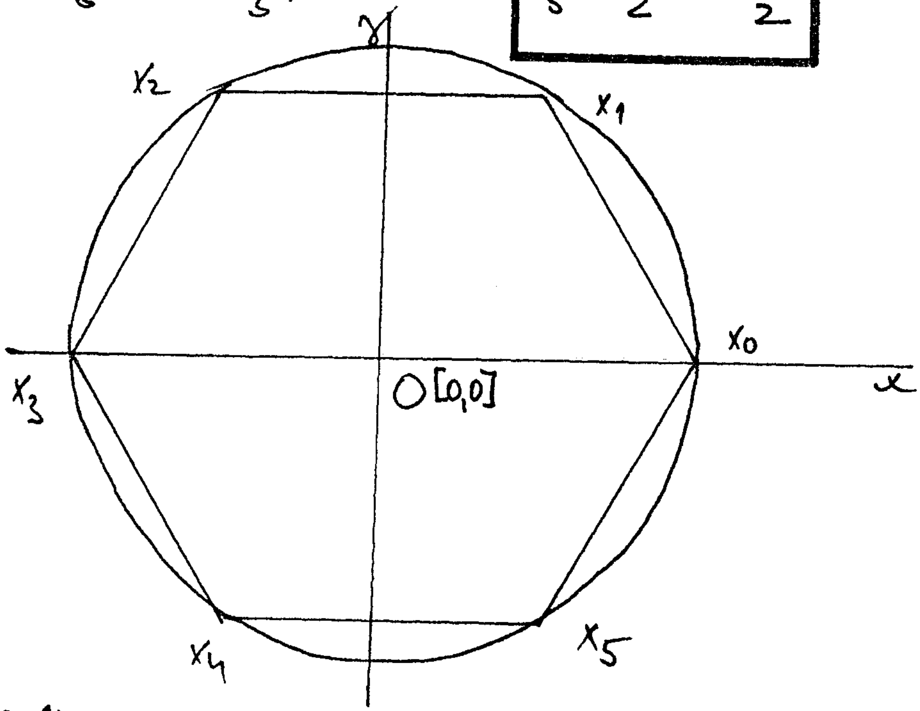
$x_3 = \cos\frac{1}{3}\pi\cdot 3 + i\sin\frac{1}{3}\pi\cdot 3$

$x_3 = \cos\pi + i\sin\pi \quad \ldots \quad \boxed{x_3 = -1}$

$x_4 = \cos\frac{1}{3}\pi\cdot 4 + i\sin\frac{1}{3}\pi\cdot 4$

$x_4 = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi \quad \ldots \quad \boxed{x_4 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}}$

$x_5 = \cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi \quad \ldots \quad \boxed{x_5 = \frac{1}{2} - i\frac{\sqrt{3}}{2}}$



2. způsob:

$x^6 - 1 = 0$

$(x^3)^2 - 1^2 = 0$

$(x^3+1)\cdot(x^3-1) = 0$

$(x^3+1^3)\cdot(x^3-1^3) = 0$

$(x+1)\cdot(x^2-x+1)\cdot(x-1)\cdot(x^2+x+1) = 0$

Jejím řešením jsou kořeny rovnice.

Pomoc: $A^3+B^3 = (A+B)\cdot(A^2-AB+B^2)$

$A^3-B^3 = (A-B)(A^2+AB+B^2)$

$x+1=0 \qquad x-1=0$

$\boxed{x=-1} \qquad \boxed{x=1}$

$x^2-x+1=0$

$$x_{1,2}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}=\begin{cases}\frac{1+i\sqrt{3}}{2}=\boxed{\frac{1}{2}+i\,\frac{\sqrt{3}}{2}}\\[4pt] \frac{1-i\sqrt{3}}{2}=\boxed{\frac{1}{2}-i\,\frac{\sqrt{3}}{2}}\end{cases}$$

$x^2+x+1=0$

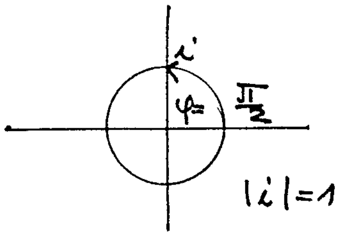
$$x_{1,2}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm i\sqrt{3}}{2}=\begin{cases}\boxed{-\frac{1}{2}+i\,\frac{\sqrt{3}}{2}}\\[4pt] \boxed{-\frac{1}{2}-i\,\frac{\sqrt{3}}{2}}\end{cases}$$

Příklad 4 (3.6184)

a) $x^3-i=0$

$x^3-(+i)=0$

$x^3-\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$



$$x_k=\underbrace{\sqrt[3]{1}}_{1}\left(\cos\frac{\frac{\pi}{2}+2k\pi}{3}+i\sin\frac{\frac{\pi}{2}+2k\pi}{3}\right),\ \text{kde } k=0,1,2$$

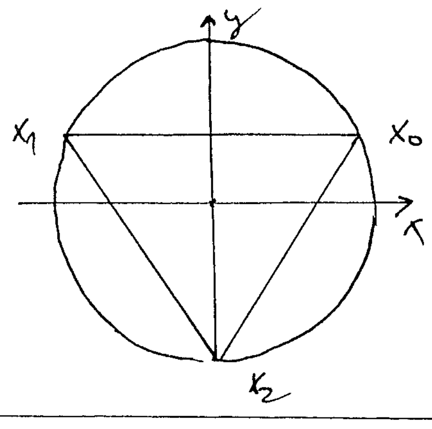
$$x_0=\cos\frac{\frac{\pi}{2}}{3}+i\sin\frac{\frac{\pi}{2}}{3}=\boxed{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}+\frac{1}{2}i=x_0}$$

$$x_1=\cos\frac{\frac{\pi}{2}+2\pi}{3}+i\sin\frac{\frac{\pi}{2}+2\pi}{3}=\boxed{\cos\frac{5}{6}\pi+i\sin\frac{5}{6}\pi=x_1}$$

$$x_2=\cos\frac{\frac{\pi}{2}+4\pi}{3}+i\sin\frac{\frac{\pi}{2}+4\pi}{3}$$

$$x_2=\frac{4\frac{1}{2}\pi}{3}+i\,\cancel{\sin}\,\frac{4\frac{1}{2}\pi}{3}$$

$$\boxed{x_2=\frac{3}{2}\pi+i\sin\frac{3}{2}\pi}$$



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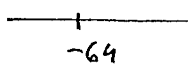
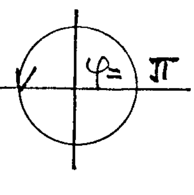
b) $x^6+64=0$

$x^6-(-64)=0$

$|-64|=64$





$x^6 - 64(\cos\pi + i\sin\pi)$

$x_k = \underbrace{\sqrt[6]{64}}_{2} \cdot \left(\cos\frac{\pi + 2k\pi}{6} + i\sin\frac{\pi + 2k\pi}{6}\right)$, kde $k = 0,1,2,3,4,5$

Hodnoty jednotlivých kořenů je možné vypočítat obdobným způsobem, např. (opět lze pomocí posun. na str. 14):

$x_0 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \boxed{\sqrt{3} + i}$

$x_1 = 2\left(\cos\frac{\pi + 2\pi}{6} + i\sin\frac{\pi + 2\pi}{6}\right) = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) =$

$= 2(0 + i\cdot 1) = \boxed{2i}$ atd.

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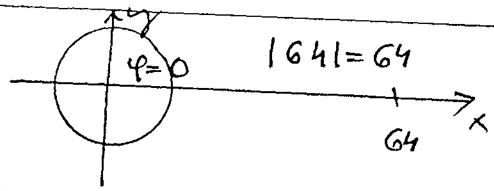
c) $x^6 - 64 = 0$

$x^6 - (+64) = 0$



$x^6 - 64(\cos 0 + i\sin 0)$

$x_k = \sqrt[6]{64}\left(\cos\frac{0 + 2k\pi}{6} + i\sin\frac{0 + 2k\pi}{6}\right)$

$x_k = 2\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right)$, kde $k = 0,1,2,3,4,5$

např.

$x_0 = 2\left(\cos\frac{0\pi}{3} + i\sin\frac{0\pi}{3}\right) = 2(\cos 0 + i\sin 0) = 2(1 + i\cdot 0) = \boxed{2}$

$x_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \boxed{1 + i\sqrt{3}}$ atd.

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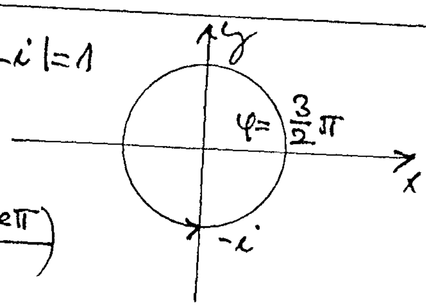
d) $x^3 + i = 0$

$x^3 - (-i) = 0$

$|-i| = 1$



$x_k = \sqrt[3]{1}\cdot\left(\cos\frac{\frac{3}{2}\pi + 2k\pi}{3} + i\sin\frac{\frac{3}{2}\pi + 2k\pi}{3}\right)$

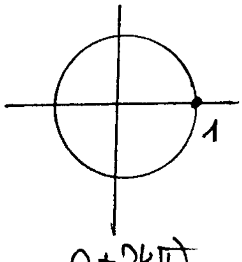
$x_k = \cos\frac{\frac{3}{2}\pi + 2k\pi}{3} + i\sin\frac{\frac{3}{2}\pi + 2k\pi}{3}$, kde $k = 0,1,2$

atd.

Příklad 5: Dále jsem použil již dříve zpracované příklady z Matice na ⑥ str. 84; čísla jsou vždy uvedena.

$\boxed{3.7/84}$

a) $x^4-1=0$



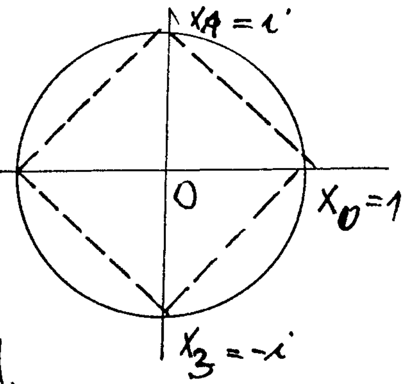
$|1|=1$

$\varphi = 0^\circ \ (0\pi = 0)$

$x^4-(+1)=0$

$$x_k = \underbrace{\sqrt[4]{1}}_{=1}\cdot\left(\cos\frac{0+2k\pi}{4} + i\sin\frac{0+2k\pi}{4}\right) = \left(\cos\frac{2k\pi}{4} + i\sin\frac{2k\pi}{4}\right)$$

$$\boxed{x_k = \left(\cos\frac{k\pi}{2} + i\sin\frac{k\pi}{2}\right),\ k=0,1,2,3}$$



$x_0 = \cos\frac{0\pi}{2} + i\sin\frac{0\pi}{2} = \cos 0 + i\sin 0 = \boxed{1}$

$x_1 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i\cdot 1 = \boxed{i}$

$x_2 = \cos\frac{2\pi}{2} + i\sin\frac{2\pi}{2} = \cos\pi + i\sin\pi = -1+0i = \boxed{-1}$

$x_3 = \cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi = 0 - 1\cdot i = \boxed{-i}$

Obrazy kořenů dané rovnice jsou vrcholy čtverce (viz obr.).

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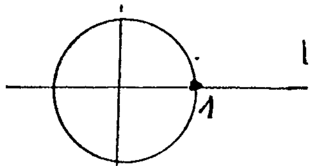
b) $x^8-1=0$



$|1|=1$

$\varphi = 0^\circ \ (0\pi = 0)$

$x^8-(+1)=0$

$$x_k = \sqrt[8]{1}\cdot\left(\cos\frac{0+2k\pi}{8} + i\sin\frac{0+2k\pi}{8}\right)$$

$$x_k = \left(\cos\frac{k\pi}{4} + i\sin\frac{k\pi}{4}\right) \ \ldots\ k = 0,1,2,3,4,5,6,7$$

$x_0 = (\cos 0 + i\sin 0) = \boxed{1}$

$x_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\sin\frac{\sqrt{2}}{2}$

$(\frac{\pi}{4} = 45^\circ)$

$x_2 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + 1i$

$= \boxed{i}$

$x_3 = \cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$\frac{3}{4}\pi = 135^\circ$

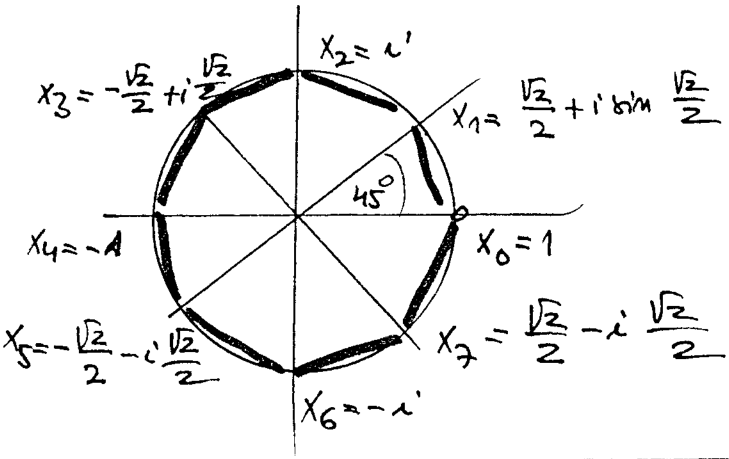
$x_4 = \cos\pi + i\sin\pi = \boxed{-1}$

$x_5 = \cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi =$

$= -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

$x_6 = \cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi = 0 - 1i = \boxed{-i}$

$x_7 = \cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$



Obraz kořenů dané rovnice jsou vrcholy pravidelného osmiúhelníku.

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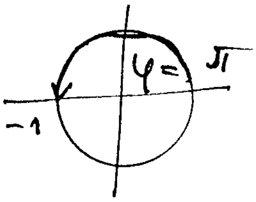
c) $x^4 + 1 = 0$

$x^4 - (-1) = 0$



$\varphi = \pi \qquad |-1| = 1$

$$x_k = \sqrt[4]{1} \cdot \left(\cos \frac{\pi + 2k\pi}{4} + i \sin \frac{\pi + 2k\pi}{4} \right)$$

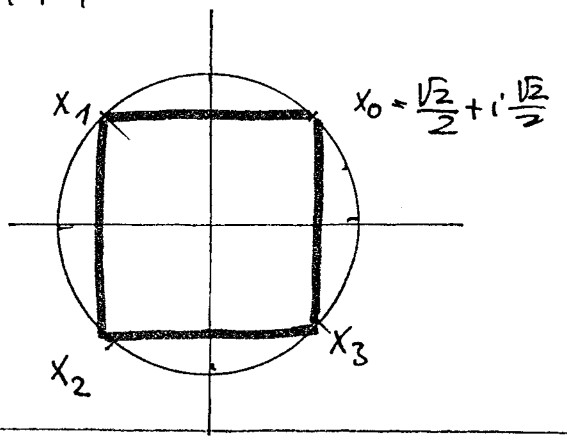
$$x_k = \left(\cos \frac{\pi + 2k\pi}{4} + i \sin \frac{\pi + 2k\pi}{4} \right) \ldots k = 0, 1, 2, 3$$

$x_0 = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ $\quad (\frac{\pi}{4} = 45^\circ$

$x_1 = \cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$x_2 = \frac{5}{4}\pi + i \sin \frac{5}{4}\pi = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

$x_3 = \frac{7}{4}\pi + i \sin \frac{7}{4}\pi = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$



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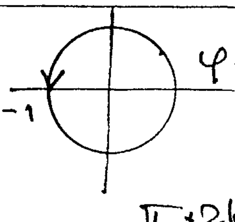
d) $x^8 - 1 = 0$

$x^8 - (-1) = 0$



$\varphi = \pi \quad |-1| = 1$

$$x_k = \sqrt[8]{1} \cdot \left(\cos \frac{\pi + 2k\pi}{8} + i \sin \frac{\pi + 2k\pi}{8} \right)$$

$$x_k = \left(\cos \frac{\pi + 2k\pi}{8} + i \sin \frac{\pi + 2k\pi}{8} \right) \ldots k = 0, 1, 2, 3, 4, 5, 6, 7$$

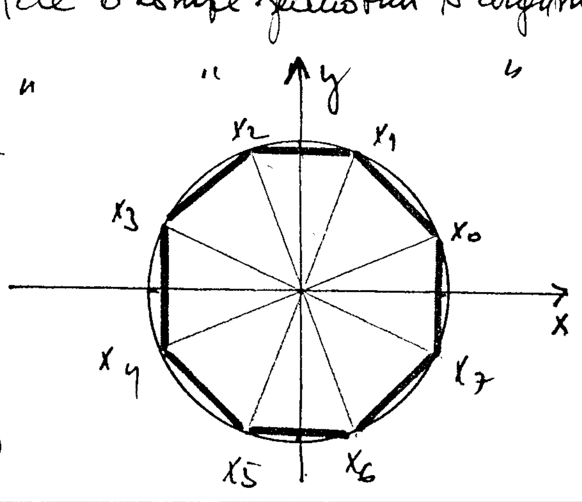
$x_0 = \cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$, jde o komplexní jednotku s argumentem $\frac{\pi}{8} = 22{,}5^\circ$

$x_1 = \cos \frac{3}{8}\pi + i \sin \frac{3}{8}\pi$, " " " $\qquad \frac{3}{8}\pi = 67{,}5^\circ$

$x_2 = \cos \frac{5}{8}\pi + i \sin \frac{5}{8}\pi \qquad \frac{5}{8}\pi = 112{,}5^\circ$

$x_3 = \cos \frac{7}{8}\pi + i \sin \frac{7}{8}\pi \qquad \frac{7}{8}\pi \to$

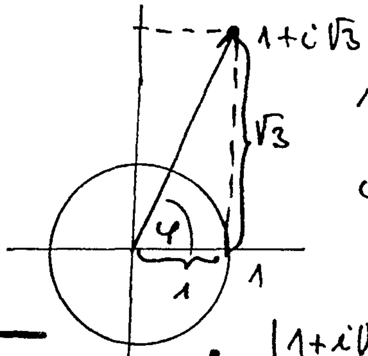
atd.



$\boxed{3.8/184}$ V C řešte rovnice:

a) $x^5-1-i\sqrt{3}=0$

$x^5-(1+i\sqrt{3})=0$



$tg\,\varphi=\frac{\sqrt{3}}{1}=\sqrt{3}$

$\varphi=\frac{\pi}{3}\,(60^\circ)$

$|1+i\sqrt{3}|=\sqrt{1^2+(\sqrt{3})^2}=$

$=\sqrt{1+3}=\sqrt{4}=2$

$$\boxed{x_k=\sqrt[5]{2}\left(\cos\frac{\frac{\pi}{3}+2k\pi}{5}+i\sin\frac{\frac{\pi}{3}+2k\pi}{5}\right),\ k=0,1,2,3,4}$$

$x_0=\sqrt[5]{2}\left(\cos\frac{\pi}{15}+i\sin\frac{\pi}{15}\right)=\sqrt[5]{2}\left(\cos 12^\circ+i\sin 12^\circ\right)$

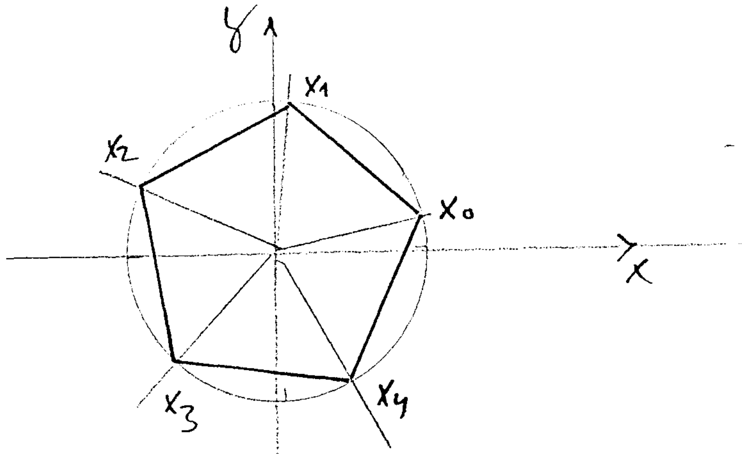
$x_1=\sqrt[5]{2}\left(\cos\frac{\frac{\pi}{3}+2\pi}{5}+i\sin\frac{\frac{\pi}{3}+2\pi}{5}\right)$

$x_1=\sqrt[5]{2}\left(\cos\frac{7}{15}\pi+i\sin\frac{7}{15}\pi\right)=\sqrt[5]{2}\cdot\left(\cos 84^\circ+i\sin 84^\circ\right)$

$x_2=\sqrt[5]{2}\left(\cos\frac{13}{15}\pi+i\sin\frac{3}{15}\pi\right)=\sqrt[5]{2}\left(\cos 156^\circ+i\sin 156^\circ\right)$

$x_3=\sqrt[5]{2}\left(\cos\frac{19}{15}\pi+i\sin\frac{19}{15}\pi\right)=\sqrt[5]{2}\left(\cos 228^\circ+i\sin 228^\circ\right)$

$x_4=\sqrt[5]{2}\left(\cos\frac{5}{3}\pi+i\sin\frac{5}{3}\pi\right)=\sqrt[5]{2}\left(\cos 300^\circ+i\sin 300^\circ\right)$



Jsem udělal nárys, abych si ověřil, jak vypadá pěti-úhelník, jehož vrcholy jsou obrazy kořenů dané rovnice.

b) už jen obdobně

$x^5+1-i\sqrt{3}=0$

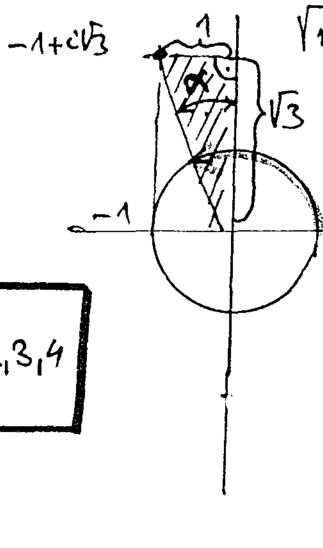
$x^5-(-1+i\sqrt{3})=0$

$tg\,\alpha=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

$\Rightarrow\alpha=\frac{\pi}{6}\,(30^\circ)$

$\varphi=90^\circ+30^\circ=120^\circ$

$\boxed{\varphi=\frac{2}{3}\pi}$



$|-1+i\sqrt{3}|=$

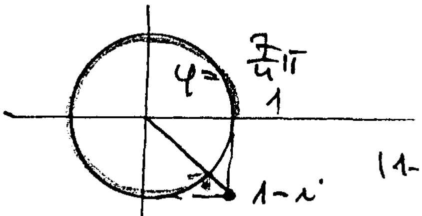
$\sqrt{1^2+(\sqrt{3})^2}=$

$=\sqrt{4}=2$

$$\boxed{x_k=\sqrt[5]{2}\cdot\cos\frac{\frac{2}{3}\pi+2k\pi}{5}+i\sin\frac{\frac{2}{3}\pi+2k\pi}{5}\ ;\ k=0,1,2,3,4}$$

c) $x^3 - 1 + i = 0$

$x^3 - (+1 - i) = 0$



$|1-i| = \sqrt{1^2+1^2} = \sqrt{2}$

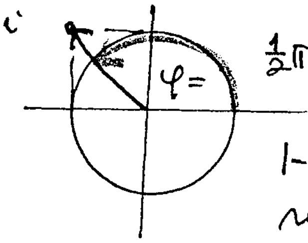
$$x_k = \sqrt[3]{\sqrt{2}}\left(\cos\frac{\frac{7}{4}\pi + 2k\pi}{3} + i\,\frac{\frac{7}{4}\pi + 2k\pi}{3}\right)$$

$(x^{\frac{1}{2}})^{\frac{1}{3}} = x^{\frac{1}{6}} = 2^{\frac{1}{6}} = \sqrt[6]{2}$

$$\boxed{x_k = \sqrt[6]{2}\left(\cos\frac{\frac{7}{4}\pi + 2k\pi}{3} + i\,\frac{\frac{7}{4}\pi + 2k\pi}{3}\right)} \quad k = 0,1,2$$

d) $x^3 + 1 - i = 0$

$x^3 - (-1 + i) = 0$



$\frac{1}{2}\pi + \frac{1}{4}\pi = \frac{3}{4}\pi$

$|-1 + i| = \sqrt{2}$ (jako výše)

$x_k = \sqrt[3]{\sqrt[2]{2}}\,(\ldots$

$$\boxed{x_k = \sqrt[6]{2}\left(\cos\frac{\frac{3}{4}\pi + 2k\pi}{3} + i\,\frac{\frac{3}{4}\pi + 2k\pi}{3}\right);\ k = 0,1,2}$$

$\boxed{3.9/84}$ v C řešte jako kvadratické rovnice a dodatečně jako binomické rovnice.

a) $x^2 - 2x + 2 = 0$

1. způsob: $x^2 - 2x + 2 = 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm i\sqrt{4}}{2} = \frac{2 \pm i \cdot 2}{2} =$$

$$= \frac{\cancel{2}(1 \pm i)}{\cancel{2}} = \begin{cases} \boxed{1+i} \\ \boxed{1-i} \end{cases}$$

2. způsob: $x^2 - 2x + 2 = 0$

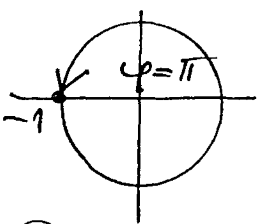
$$\underbrace{x^2 - 2x + 1}_{} + 1 = 0$$

$$\underbrace{(x-1)}^2 + 1 = 0$$

substituce: $x - 1 = z$

$z^2 + 1 = 0$

$z^2 - (-1) = 0$



$|-1| = 1$

$$z_k=\sqrt[2]{1}\cdot\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)$$

$$z_k=\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)\ \ldots k=0,1$$

$$z_0=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+i\cdot 1=\boxed{i},\quad x_0-1=z_0$$

$$x_0-1=i$$

$$\boxed{x_0=1+i}$$

$$z_1=\cos\frac{\pi+2\pi}{2}+i\sin\frac{\pi+2\pi}{2}=\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi$$

$$z_1=0-1\cdot i=\boxed{-i}\ \ldots\ x_1-1=z_1$$

$$x_1-1=-i$$

$$\boxed{x_1=1-i}$$

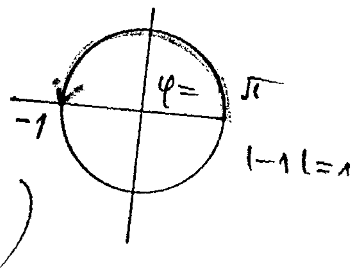
b) $x^2+1=0$

1. spôsob: $x^2+1=0$

$$x^2=-1$$

$$x=\pm\sqrt{-1}\ \begin{cases}\boxed{i}\\ \boxed{-i}\end{cases}$$

2. spôsob: $x^2-(-1)=0$



$$x_k=\sqrt{1}\cdot\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)$$

$$x_k=\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\ \ldots k=0,1$$

$$x_0=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+i\cdot 1=\boxed{i}$$

$$x_1=\cos\frac{\pi+2\pi}{2}+i\sin\frac{\pi+2\pi}{2}=\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi=0-i=\boxed{-i}$$

c) $x^2-2x+4=0$

1. způsob: $x^2-2x+4=0$

$$x_{1,2}=\frac{2\pm\sqrt{4-16}}{2}=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm i\sqrt{12}}{2}=\frac{2\pm i\cdot 2\sqrt{3}}{2}$$

$$x_{1,2}=\frac{\cancel{2}(1\pm i\sqrt{3})}{\cancel{2}}=\begin{cases}\boxed{x_1=1+i\sqrt{3}}\\ \boxed{x_2=1-i\sqrt{3}}\end{cases}$$

2. způsob: $x^2-2x+4=0$

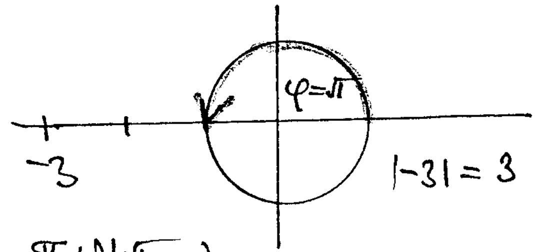
$$\underbrace{x^2-2x+1}+3=0$$

$$\underbrace{(x-1)}^2+3=0$$

Substituce $x-1=z$

$$z^2+3=0$$

$$z^2-(-3)=0$$



$$z_k=\sqrt{3}\cdot\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)\quad\dots k=0,1$$

$$z_0=\sqrt{3}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=\sqrt{3}\,(0+i\cdot 1)=\sqrt{3}\cdot i\ \dots\ x_0-1=z_0$$

$$x_0-1=i\sqrt{3}$$

$$\boxed{x_0=1+i\sqrt{3}}$$

$$z_1=\sqrt{3}\left(\cos\frac{\pi+2\pi}{2}+i\sin\frac{\pi+2\pi}{2}\right)$$

$$z_1=\sqrt{3}\left(\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi\right)=\sqrt{3}\cdot(0-1\cdot i)=\sqrt{3}\cdot(-i)=-i\sqrt{3}$$

$$x_1-1=z_1$$

$$x_1-1=-i\sqrt{3}$$

$$\boxed{x_1=1-i\sqrt{3}}$$

---

d) $x^2+2x+2=0$

1. způsob: $x^2+2x+2=0$

$$x_{1,2}=\frac{-2\pm\sqrt{4-8}}{2}=\frac{-2\pm\sqrt{-4}}{2}=\frac{-2\pm i\cdot\sqrt{4}}{2}=\frac{-2\pm 2i}{2}$$

$$=\frac{2(-1\pm i)}{2}=-1\pm i=\begin{cases}\boxed{-1+i}\\ \boxed{-1-i}\end{cases}$$

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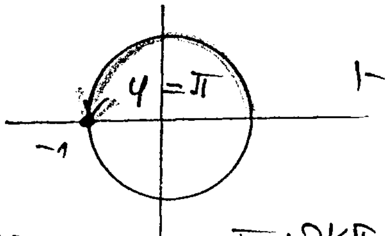
2. způsob: $x^2+2x+2=0$ $\qquad \ldots \underbrace{x^2+2x+1}+1=0$

$\underbrace{(x+1)}^2+1=0$ $\qquad (x+1)^2+1=0$

subst. $x+1=z$

$z^2+1=0$

$z^2-(-1)=0$



$|-1|=1$

$$z_k=\sqrt{1}\cdot\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)$$

$$z_k=\left(\cos\frac{\pi+2k\pi}{2}+i\sin\frac{\pi+2k\pi}{2}\right)\ldots k=0,1$$

$z_0=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+i\cdot1=\boxed{i}$

$x_0+1=z_0$

$x_0+1=i$

$\boxed{x_0=-1+i}$

$z_1=\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi$

$z_1=0+i\cdot(-1)=\boxed{-i}$

$x_1+1=z_1$

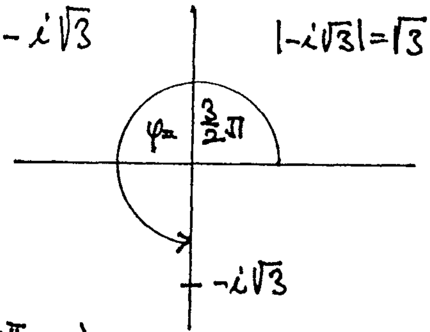
$x_1+1=-i$

$\boxed{x_1=-1-i}$

Příklady z jiných zdrojů

Příklad 6. Řešte v C rovnici $z^6=-i\sqrt{3}$

$|-i\sqrt{3}|=\sqrt{3}$



$z^6+i\sqrt{3}=0$

$z^6-(-i\sqrt{3})=0$

$z^6-\sqrt{3}(\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi)$

$$z_k=\sqrt[6]{\sqrt[2]{3}}\cdot\left(\cos\frac{\frac{3}{2}\pi+2k\pi}{6}+i\sin\frac{\frac{3}{2}\pi+2k\pi}{6}\right)$$

$$z_k=\sqrt[12]{3}\cdot\left(\cos\frac{\frac{3}{2}\pi+2k\pi}{6}+i\sin\frac{\frac{3}{2}\pi+2k\pi}{6}\right);\ k=0,1,2,3,4,5$$

$z_0=\sqrt[12]{3}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$	$z_3=\sqrt[12]{3}\left(\cos\frac{5}{4}\pi+i\sin\frac{5}{4}\pi\right)$
$z_1=\sqrt[12]{3}\left(\cos\frac{7}{12}\pi+i\sin\frac{7}{12}\pi\right)$	$z_4=\sqrt[12]{3}\left(\cos\frac{19}{12}\pi+i\sin\frac{19}{12}\pi\right)$
$z_2=\sqrt[12]{3}\left(\cos\frac{11}{12}\pi+i\sin\frac{11}{12}\pi\right)$	$z_5=\sqrt[12]{3}\left(\cos\frac{23}{12}\pi+i\sin\frac{23}{12}\pi\right)$

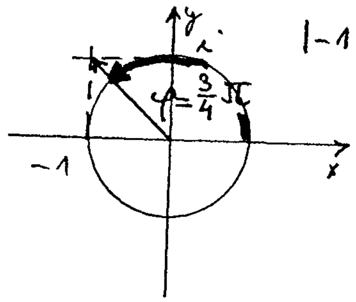
Příklad 7: Řešte v C rovnici

a) $z^4 = -1 + i$

$z^4 + 1 - i = 0$

$z^4 - (-1 + i) = 0$

$z^4 - \sqrt{2}(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi)$



$|-1+i| = \sqrt{2}$

$z_k = \underbrace{\sqrt[4]{\sqrt[2]{2}}}_{\sqrt[8]{2}} \cdot \left(\cos \frac{\frac{3}{4}\pi + 2k\pi}{4} + i \sin \frac{\frac{3}{4}\pi + 2k\pi}{4}\right)$, $k = 0, 1, 2, 3$

$z_0 = \sqrt[8]{2} \cdot \left(\cos \frac{3}{16}\pi + i \sin \frac{3}{16}\pi\right)$	$z_1 = \sqrt[8]{2} \left(\cos \frac{11}{16}\pi + i \sin \frac{11}{16}\pi\right)$
$z_2 = \sqrt[8]{2} \cdot \left(\cos \frac{19}{16}\pi + i \sin \frac{19}{16}\pi\right)$	$z_3 = \sqrt[8]{2} \left(\cos \frac{27}{16}\pi + i \sin \frac{27}{16}\pi\right)$

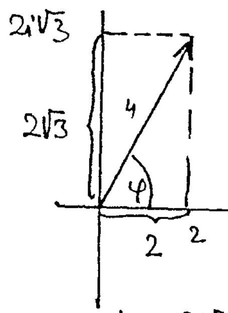
---

b) $x^5 = \underbrace{2 + 2i\sqrt{3}}_{a}$

$a = 2 + 2i\sqrt{3} = 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

$x^5 - 2 - 2i\sqrt{3} = 0$

$x^5 - (\underbrace{2 + 2i\sqrt{3}}_{a}) = 0$



φ lze určit a) z paměti (pokud vím, že $\sqrt{3} \to \varphi = \frac{\pi}{3}$), nebo $\operatorname{tg}\varphi = \frac{2\sqrt{3}}{2} \sqrt{3}$

$\cos\varphi = \frac{2}{4} = \frac{1}{2} \to \varphi = \frac{\pi}{3}$

lze pomocí $\sin\varphi$

$|2 + 2i\sqrt{3}| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{16} = 4$

$x_k = \sqrt[5]{4} \cdot \left(\cos \frac{\frac{\pi}{3} + 2k\pi}{5} + i \sin \frac{\frac{\pi}{3} + 2k\pi}{5}\right)$, kde $k = 0, 1, 2, 3, 4$

$x_0 = \sqrt[5]{4} \left(\cos \frac{\pi}{15} + i \sin \frac{\pi}{15}\right)$ $\quad x_1 = \sqrt[5]{4} \left(\cos \frac{7}{15}\pi + i \sin \frac{7}{15}\pi\right)$

$x_2 = \sqrt[5]{4} \left(\cos \frac{13}{15}\pi + i \sin \frac{13}{15}\pi\right)$ $\quad x_3 = \sqrt[5]{4} \left(\cos \frac{19}{15}\pi + i \sin \frac{19}{15}\pi\right)$

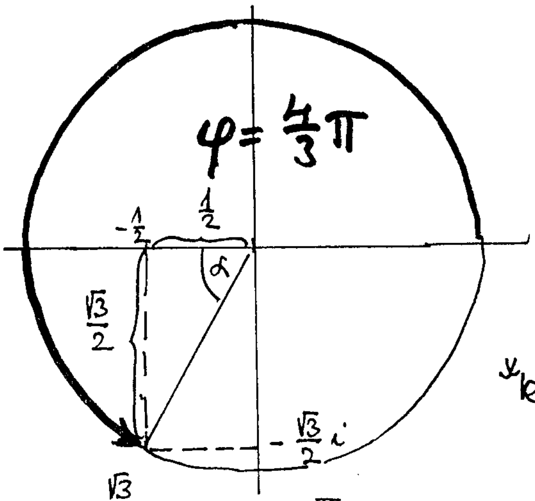
$x_4 = \sqrt[5]{4} \left(\cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi\right)$

Protože jde o $n=5$ (pětiúhelník)

Důležité poznámky: Po vypočtu $x_0 \ldots \frac{\pi}{15}$, tak pak to lze dělat přičítáním $\frac{1}{5}$ velikosti 2π, tj. $\frac{1}{5}$ ze $2\pi = \frac{1}{5} \cdot 2\pi = \frac{2}{5}\pi$

Úhel pro x_1 je: $\frac{1}{15}\pi + \frac{2}{5}\pi = \frac{7}{15}\pi$; $\frac{7}{15}\pi + \frac{2}{5}\pi = \frac{13}{15}\pi$;

$\frac{13}{15}\pi + \frac{2}{5}\pi = \frac{19}{15}\pi$; $\frac{19}{15}\pi + \frac{2}{5}\pi = \frac{5}{3}\pi$



$$\operatorname{tg}\alpha=\frac{\frac{\sqrt3}{2}}{\frac12}=\sqrt3 \Rightarrow \alpha=\frac{\pi}{3}$$

$$\varphi=\pi+\frac{\pi}{3}=\frac43\pi$$

c) $x^4=-\frac12-i\,\frac12\sqrt3$

$$x^4+\frac12+i\,\frac{\sqrt3}{2}=0$$

$$x^4-\left(-\frac12-i\,\frac{\sqrt3}{2}\right)=0$$

$$x_k=\sqrt[4]{1}\cdot\left(\cos\frac{\frac43\pi+2k\pi}{4}+i\sin\frac{\frac43\pi+2k\pi}{4}\right)$$

$$x_k=\cos\left(\frac{\frac43\pi}{4}+\frac{2k\pi}{4}\right)+i\sin\left(\frac{\frac43\pi}{4}+\frac{2k\pi}{4}\right)$$

$$x_k=\cos\left(\frac13\pi+\frac12k\pi\right)+i\sin\left(\frac13\pi+\frac12k\pi\right)$$

$2\pi:4=\frac12\pi$, podle poznámky na str. 14 určíme další hodnoty podle návodu:

$\frac13\pi+\frac12\pi=\frac56\pi$, $\frac56\pi+\frac12\pi=\frac43\pi$,

$\frac43\pi+\frac12\pi=\frac{11}{6}\pi$

$x_0=\cos\frac13\pi+i\sin\frac13\pi$ $x_0=\frac12+i\frac{\sqrt3}{2}$
$x_1=\cos\frac56\pi+i\sin\frac56\pi$ $x_1=-\frac{\sqrt3}{2}+\frac12 i$
$x_2=\cos\frac43\pi+i\sin\frac43\pi$ $x_2=-\frac12-i\frac{\sqrt3}{2}$
$x_3=\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi$ $x_3=\frac{\sqrt3}{2}-i\frac12$

<u>Příklad 8</u> (navíc): Řešte rovnici

$$(3-4i)^2-2\bar{x}=x$$

$$9-24i-16-2\bar{x}=x$$

$$-7-24i-2\bar{x}=x\quad,\ \text{Sub.: } x=a+bi,\ \bar{x}=a-bi$$

$$-7-24i-2(a-bi)=a+bi$$

$$-7-24i-2a+2bi=a+bi$$

$$-7-24i-3a+bi=0$$

$$-7-24i=3a-bi$$

$$-7=3a \qquad -24i=-bi$$

$$a=-\frac73 \qquad b=24$$

$$\boxed{x=-\frac73+24i}$$