4.7 Elipsa

① Napište rovnici elipsy v osovém tvaru, je-li:

a) $a=5, b=2$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\boxed{\frac{x^2}{25}+\frac{y^2}{4}=1}$$

b) $\boxed{\frac{x^2}{49}+\frac{y^2}{25}=1}$ $a=7, b=5$

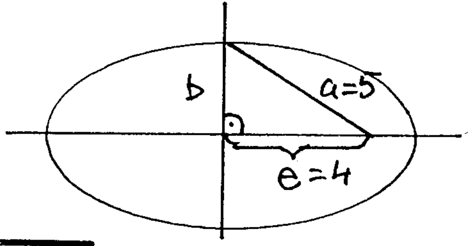
c) $a=5, e=4$

$b^2=a^2-e^2$

$b^2=25-16$

$b^2=9$

$$\boxed{\frac{x^2}{25}+\frac{y^2}{9}=1}$$



d) $a+b=18, e=12$

$a^2-b^2=e^2$

$a^2-b^2=144$

$(a+b)\cdot(a-b)=144$

$18\cdot(a-b)=144$

$a-b=8$

$a+b=18$

$\underline{a-b=8}$

$2a=26$

$a=13$

$a^2=169$

$13+b=18$

$b=5$

$b^2=25$

$$\boxed{\frac{x^2}{169}+\frac{y^2}{25}=1}$$

---

② Určete hlavní a vedlejší poloosu, lineární excentricitu a souřadnice ohnisek elipsy:

a) $\frac{x^2}{9}+\frac{y^2}{4}=1$

$a^2=9$ $b^2=4$

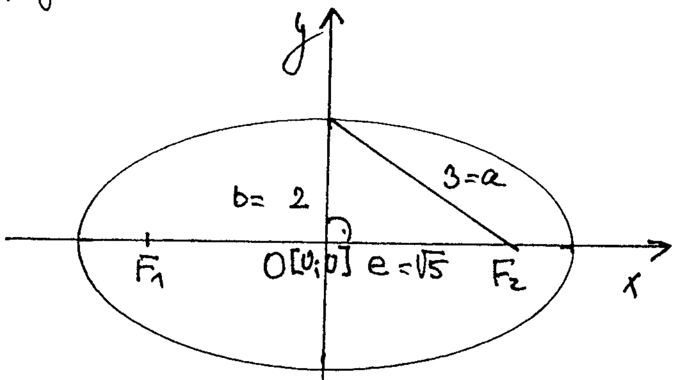
$\boxed{a=3}$ $\boxed{b=2}$

$e^2=9-4$

$e^2=5$

$\boxed{e=\sqrt{5}}$

$\boxed{F_1[-\sqrt{5};0], F_2[\sqrt{5};0]}$



b) $\frac{x^2}{32}+\frac{y^2}{16}=1$

$a=\sqrt{32}$ $b^2=\sqrt{16}$

$\boxed{a=4.\sqrt{2}}$ $\boxed{b=4}$

$e^2=a^2-b^2$

$e^2=(\sqrt{32})^2-16$

$e^2=32-16$

$e^2=16$

$\boxed{e=4}$

$\boxed{F_1[-4;0]}$

$\boxed{F_2[4;0]}$

c) $9x^2-16y^2=144$ $|\cdot\frac{1}{144}$

$\frac{x^2}{16}-\frac{y^2}{9}=1$

$\boxed{a=4}$

$\boxed{b=3}$

$e^2=16-9$

$e^2=7$

$\boxed{e=\sqrt{7}}$

$\boxed{F_1[-\sqrt{7};0]}$

$\boxed{F_2[\sqrt{7};0]}$

d) $16x^2+25y^2=400 \quad | \cdot \frac{1}{400}$

$\frac{x^2}{25}+\frac{y^2}{16}=1$

$\boxed{a=5}$

$\boxed{b=4}$

$e^2=a^2-b^2$

$e^2=25-16$

$e^2=9 \rightarrow \boxed{e=3}$

$\boxed{\begin{array}{l} F_1[-3;0] \\ F_2[3;0] \end{array}}$

---

③ Napište rovnici elipsy v osovém tvaru, která prochází body:

a) $A=[6;4]$, $B[8;3]$

Do rovnice $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ dosadíme postupně souřadnice bodů A, B.

A: $\frac{36}{a^2}+\frac{16}{b^2}=1 \quad | \cdot a^2b^2$

B: $\frac{64}{a^2}+\frac{9}{b^2}=1 \quad | \cdot a^2b^2$

$\boxed{1}$ $36b^2+16a^2=a^2b^2 \quad | \cdot(-1)$

$64b^2+9a^2=a^2b^2$ $\boxed{2}$

$-36b^2-16a^2=-a^2b^2$

$28b^2-9a^2=0$

$9a^2=28b^2 \quad | \cdot \frac{1}{7}$

$\boxed{a^2=4b^2}$ dosadíme do $\boxed{1}$

$36b^2+16\cdot 4b^2=4b^2\cdot b^2$

$36b^2+64b^2=4b^4$

$4b^4=100b^2 \quad | : 4b^2$

$\boxed{b^2=25} \longrightarrow$

$a^2=4\cdot 25$

$\boxed{a^2=100}$

$\boxed{\frac{x^2}{100}+\frac{y^2}{25}=1}$

---

b) $A[1;3]$, $B[3;2]$

A: $\frac{1}{a^2}+\frac{9}{b^2}=1 \quad | \cdot a^2b^2$

B: $\frac{9}{a^2}+\frac{4}{b^2}=1 \quad | \cdot a^2b^2$

$\boxed{1}$ $\boxed{b^2+9a^2=a^2b^2} \quad | \cdot(-1)$

$\boxed{2}$ $9b^2+4a^2=a^2b^2$

$-b^2-9a^2=-a^2b^2$

$8b^2-5a^2=0$

$8b^2=5a^2$

$\boxed{b^2=\frac{5}{8}a^2}$ dej do $\boxed{1}$

$\frac{5}{8}a^2+9a^2=a^2\cdot\frac{5}{8}a^2 \quad | \cdot 8$

$5a^2+72a^2=5a^4$

$5a^4=77a^2 \quad | : a^2$

$5a^2=77$

$\boxed{a^2=\frac{77}{5}}$

$b^2=\frac{5}{8}\cdot\frac{77}{5}$

$\boxed{b^2=\frac{77}{8}}$

$\frac{x^2}{\frac{77}{5}}+\frac{y^2}{\frac{77}{8}}=1 \rightarrow \boxed{\frac{5x^2}{77}+\frac{8y^2}{77}=1}$

---

c) $A[-4;4]$, $B[\sqrt{5};-7]$

$A \dots \frac{16}{a^2}+\frac{16}{b^2}=1$

$\boxed{1}$ $\boxed{16b^2+16a^2=a^2b^2}$ $| \cdot(-1)$

$B: \frac{(\sqrt{5})^2}{a^2}+\frac{49}{b^2}=1$

$\boxed{2}$ $\boxed{\frac{5}{a^2}+\frac{49}{b^2}=a^2b^2}$ ‡

$\boxed{5b^2+49a^2=a^2b^2}$

$-16b^2-16a^2=-a^2b^2$

$-11b^2+33a^2=0$

$11b^2=33a^2$

$\boxed{b^2=3a^2}$ daj do $\boxed{1}$

$16\cdot 3a^2+16a^2=a^2\cdot 3a^2$

$64a^2=3a^4 \quad | :a^2$

$64=3a^2$

$\boxed{a^2=\frac{64}{3}}$

$b^2=3\cdot\frac{64}{3}$

$\boxed{b^2=64}$

$\frac{x^2}{\frac{64}{3}}+\frac{y^2}{64}=1$

$\frac{3x^2}{64}+\frac{y^2}{64}=1$

---

d) $A[2;3], B[-1;-4]$ $\quad B \dots \frac{1}{a^2}+\frac{16}{b^2}=1$

$A \dots \frac{4}{a^2}+\frac{9}{b^2}=1$

$b^2+16a^2=a^2b^2$

$4b^2+9a^2=a^2b^2 \quad | \cdot(-1) \longrightarrow -4b^2-9a^2=-a^2b^2$

$-3b^2+7a^2=0$

$3b^2=7a^2$

$b^2=\frac{7}{3}a^2$

$4\cdot\frac{7}{3}a^2+9a^2=a^2\cdot\frac{7}{3}a^2$

$\frac{28}{3}a^2+9a^2=\frac{7}{3}a^4 \quad | \cdot 3$

$28a^2+27a^2=7a^4$

$55a^2=7a^4 \quad | :a^2$

$55=7a^2$

$a^2=\frac{55}{7}$

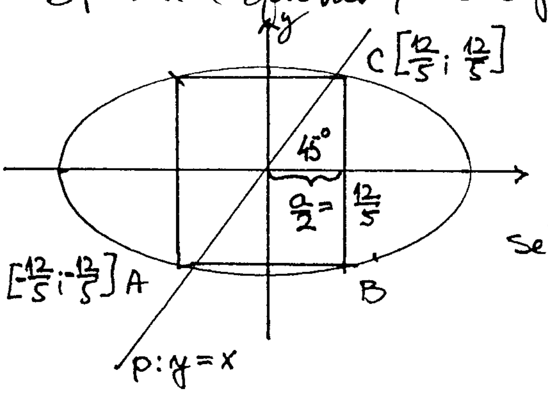
$b^2=\frac{7}{3}\cdot\frac{55}{7}$

$b^2=\frac{55}{3}$

$\frac{x^2}{\frac{55}{7}}+\frac{y^2}{\frac{55}{3}}=1 \longrightarrow \boxed{\frac{7x^2}{55}+\frac{3y^2}{55}=1}$

---

* ④ Elipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ je vpísaný štvorec. Vypočítajte jeho stranu (správne má byť: dĺžku jeho strany).



Priamka p: y = x obsahuje uhlopriečku AC, s osou x zviera uhol 45°.

Najskôr vypočítame súradnice priesečníkov A, C – t.j. priamky a elipsy.

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$y=x$$

$$\frac{x^2}{16}+\frac{x^2}{9}=1$$

$$\left(\frac{1}{16}+\frac{1}{9}\right)x^2=1$$

$$\frac{25}{144}x^2=1$$

$$25x^2=144$$

$$x^2=\frac{144}{25}$$

$$x_{1,2}=\pm\frac{12}{5}$$

x se rovná $\frac{a}{2}$

proto

$$a=2x=2\cdot\frac{12}{5}$$

$$\boxed{a=\frac{24}{5}}$$

*⑤ Na elipse $\frac{x^2}{100}+\frac{y^2}{36}=1$ najděte bod, jehož vzdálenost od jednoho ohniska elipsy je 4krát větší než od ohniska druhého.

Základem řešení je využití definice elipsy: V rovině jsou dány dva různé body F_1, F_2. Množina všech bodů X roviny, pro které se součet $|XF_1|+|XF_2|$ vzdáleností bodu X od bodů F_1, F_2 rovná danému číslu většímu než $|F_1F_2|$, se nazývá elipsa.
Hledaný bod elipsy označme $X[x;y]$

$$\frac{x^2}{100}+\frac{y^2}{36}=1 \quad \Big| \quad a^2=100,\ a=10 \quad \Big| \quad b^2=36,\ b=6$$

$e^2=a^2-b^2$

$e^2=100-36$

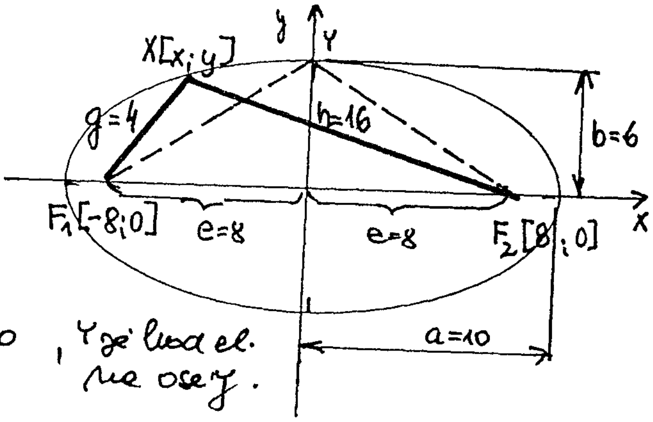
$e^2=64$

$e=8$

$F_1[-8;0]$, $F_2[8;0]$

z definice plyne:

$|F_1Y|+|F_2Y|=2a=2.10=20$, Y je bod el. na ose y.



Označme:

$g=|F_1X|$, $h=|F_2X|$

$g+h=2a$, neboť bod X je bodem elipsy.

$$\left.\begin{aligned} g+h&=20\\ h&=4g \end{aligned}\right\}$$ Řešme tuto soustavu rovnic.

$g+4g=20$

$5g=20$

$g=4$ nebo též $|\vec{g}|=4$

$h=4g$

$h=4.4$

$h=16$ nebo též $|\vec{h}|=16$

$\vec{g}=X-F_1=(x+8,\ y-0)=(x+8;y)$

$4^2=(x+8)^2+y^2$

$16=x^2+16x+64+y^2$

$\boxed{1}$ $\boxed{x^2+y^2+16x+48=0}$ $\quad |.(-1)$

$\vec{n} = X - F_2 = (x-8\ ;\ y-0)$

$\vec{n} = (x-8\,;\,y)$

$16^2 = (x-8)^2 + y^2$

$256 = x^2 - 16x + 64 + y^2$

$\boxed{2}\ \boxed{x^2 + y^2 - 16x - 192 = 0}$ připoj $\boxed{1}$ $/\cdot(-1)$

$-x^2 - y^2 - 16x - 48 = 0$

$-32x = 240$

$x = -\frac{15}{2}\ (-7{,}5)$

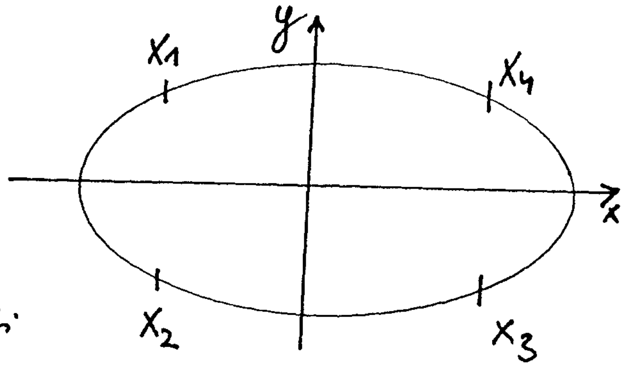
Dej do $\boxed{2}$

$\left(-\frac{15}{2}\right)^2 + y^2 - 16\left(-\frac{15}{2}\right) - 192 = 0$

$\frac{225}{4} + y^2 + 120 - 192 = 0$

$y^2 = \frac{63}{4} = \frac{9\cdot 7}{4}$

$y_1 = \frac{3\cdot\sqrt{7}}{2}\ ,\ y_2 = -\frac{3\cdot\sqrt{7}}{2}$



Vzhledem k souměrnosti elipsy podle os x, y má úloha 4 řešení. $X_1\left[-\frac{15}{2}; \frac{3\cdot\sqrt{7}}{2}\right]$, $X_2\left[-\frac{15}{2}; -\frac{3\sqrt{7}}{2}\right]$, $X_3\left[\frac{15}{2}; -\frac{3\sqrt{7}}{2}\right]$, $X_4\left[\frac{15}{2}; \frac{3\sqrt{7}}{2}\right]$

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⑥ Určete vzájemnou polohu přímky a elipsy:

a) $2x + y - 5 = 0 \qquad \frac{x^2}{16} + \frac{y^2}{9} = 1$

Předběžná úvaha: Při řešení dospějeme ke kvadratické rovnici. Bude-li její $D > 0$, budou existovat 2 průsečíky přímky s elipsou. My však pro ilustraci budeme ve výpočtu pokračovat.

$2x + y - 5 = 0$

$y = -2x + 5$

x	0	2
y	5	1

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow a = 4\ ,\ b = 3$

$\frac{x^2}{16} + \frac{(-2x+5)^2}{9} = 1$

$\frac{x^2}{16} + \frac{4x^2 - 20x + 25}{9} = 1 \quad |\cdot 144$

$9x^2 + 64x^2 - 320x + 400 = 144$

$73x^2 - 320x + 256 = 0 \quad \rightarrow D = 27648 \Rightarrow$

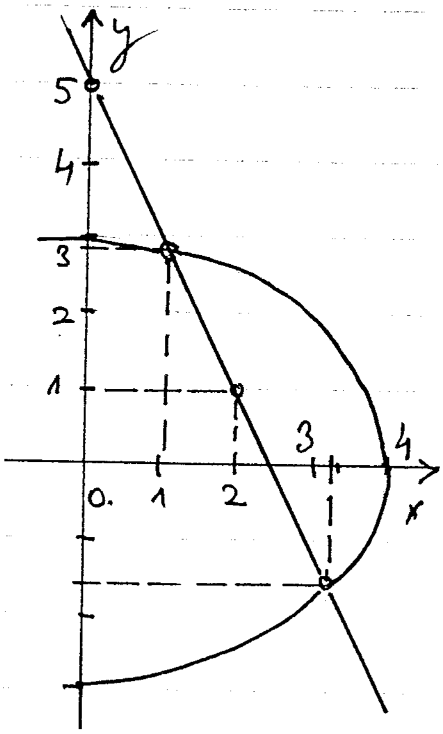
Přímka je sečnou elipsy. My však budeme

pokračovat

$$x_{1,2}=\frac{320\pm\sqrt{27648}}{146}=\frac{320\pm 96\sqrt{3}}{146}=\frac{160\pm 48\sqrt{3}}{73}=\begin{cases} x_1 \doteq 3{,}33 \\ x_2 \doteq 1{,}053 \end{cases}$$

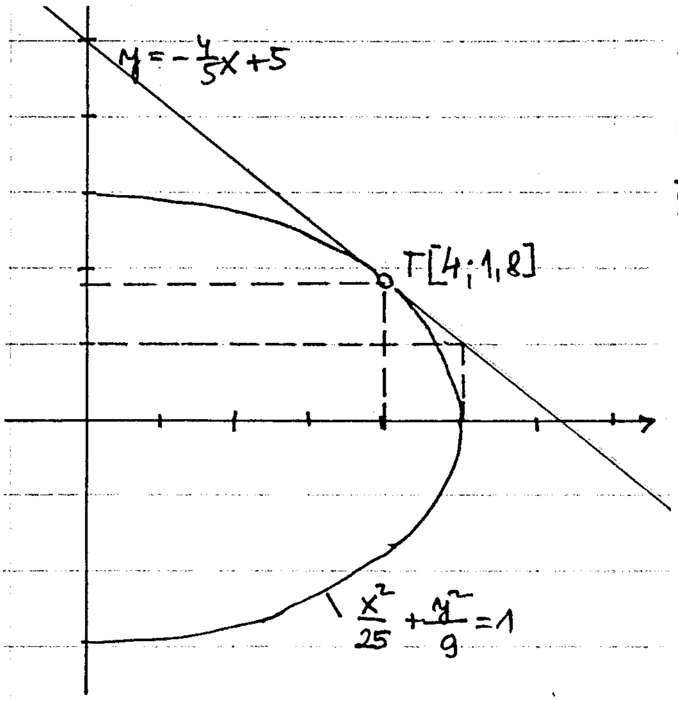
$y_1 \doteq -2 \cdot 3{,}33+5 \doteq -1{,}64$ $\quad A[3{,}33; -1{,}64]$

$y_2 = -2 \cdot 1{,}053+5 \doteq 2{,}89$ $\quad B[1{,}053; 2{,}89]$



Přímka je sečnou elipsy.

b) $4x+5y-25=0 \qquad \frac{x^2}{25}+\frac{y^2}{9}=1$



$5y=-4x+25$

$y=-\frac{4}{5}x+5$

x	0	5
y	5	1

Řešení provedeno komplexní, i když by stačilo zjistit, že $D=0$. $D=\sqrt{64-64}=0 \Rightarrow$ že jde o tečnu

$$\frac{x^2}{25}+\frac{\left(-\frac{4}{5}x+5\right)^2}{9}=1$$

$$\frac{x^2}{25}+\frac{\frac{16}{25}x^2-8x+25}{9}=1 \quad / \cdot 225$$

$$9x^2+25\cdot\left(\frac{16}{25}x^2-8x+25\right)=225$$

$$9x^2+16x^2-200x+625-225=0$$

$$25x^2-200x+400=0 \quad /:25$$

$$x^2-8x+16=0$$

$$x_{1,2}=\frac{8\pm\sqrt{64-64}}{2}=\frac{8\pm 0}{2}=4$$

$5y=-4\cdot 4+25$ $\qquad T[4;1,8]$

$y=\frac{9}{5}(1{,}8)$ $\qquad$ Bod dotyku

Přímka je tečnou elipsy.

c) $5x+4y-30=0 \qquad \frac{x^2}{9}+\frac{y^2}{4}=1$

$4y=-5x+30$

$y=-\frac{5}{4}x+\frac{15}{2}$

$$\frac{x^2}{9}+\frac{\left(-\frac{5}{4}x+\frac{15}{2}\right)^2}{4}=1$$

$$\frac{x^2}{9} + \frac{\frac{25}{16}x^2 - \frac{75}{4}x + \frac{225}{4}}{4} = 1 \quad |\cdot 36$$

$$4x^2 + 9\cdot\left(\frac{25}{16}x^2 - \frac{75}{4}x + \frac{225}{4}\right) = 36$$

$$4x^2 + \frac{225}{16}x^2 - \frac{675}{4}x + \frac{2025}{4} = 36 \quad |\cdot 16$$

$$64x^2 + 225x^2 - 2700x + 8100 - 576 = 0$$

$$\underbrace{289x^2 - 2700x + 7524 = 0}$$

Tato rovnice nemá řešení, neboť $D<0$.

$D = b^2 - 4ac$

$D = (-2700)^2 - 4\cdot 289 \cdot 7524$

$D = -1407744$

$D<0 \Rightarrow$ Daná přímka je vnější přímkou elipsy.

(viz ilustrace)

d) $4x - 5y - 40 = 0 \qquad \frac{x^2}{50} + \frac{y^2}{32} = 1$

$y = \frac{4}{5}x - 8$

$$\frac{x^2}{50} + \frac{\frac{16}{25}x^2 - \frac{64}{5}x + 64}{32} = 1 \quad |\cdot 1600$$

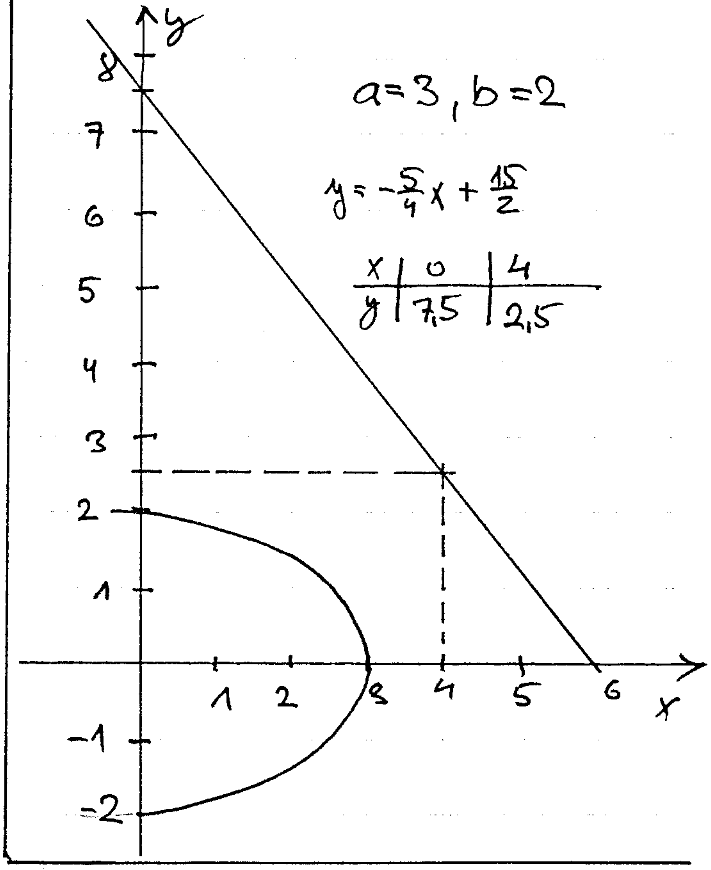
$$32x^2 + 32x^2 - 640x + 3200 - 1600 = 0$$

$$64x^2 - 640x + 1600 = 0 \quad |:64$$

$$x^2 - 10x + \overset{25}{\cancel{1600}} = 0 \qquad D=0 \rightarrow \text{tečna}$$

$$x_{1,2} = \frac{10 \pm \sqrt{100-100}}{2} = \frac{10\pm 0}{2} = 5$$

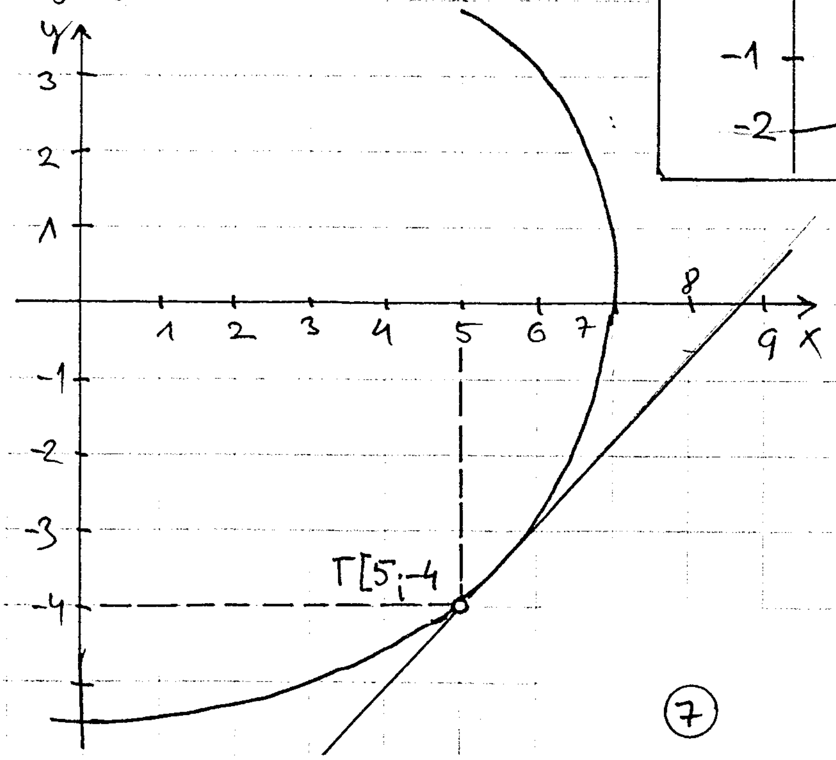
$y = \frac{4}{5}\cdot 5 - 8 = -4 \qquad T[5;-4]$



$a=3,\ b=2$

$y = -\frac{5}{4}x + \frac{15}{2}$

x	0	4
y	7,5	2,5



$a^2 = 50,\ a = \sqrt{50} = 5\cdot\sqrt{2}\ (\doteq 7{,}07)$

$b^2 = 32,\ b = \sqrt{32} = 4\sqrt{2}\ (\doteq 5{,}66)$

Přímka je tečnou elipsy.

⑦ Určete číslo c tak, aby přímka $2x+3y+c=0$ byla tečnou elipsy $\frac{x^2}{50}+\frac{y^2}{30}=1$

$$2x+3y+c=0$$
$$3y=-2x-c$$
$$y=-\frac{2}{3}x-\frac{1}{3}c$$

$$\frac{x^2}{50}+\frac{y^2}{30}=1$$
$$\frac{x^2}{50}+\frac{\left(-\frac{2}{3}x-\frac{1}{3}c\right)^2}{30}=1$$
$$\frac{x^2}{50}+\frac{\frac{4}{9}x^2+\frac{4}{9}cx+\frac{1}{9}c^2}{30}=1 \quad |\cdot 150$$
$$3x^2+5\cdot\left(\frac{4}{9}x^2+\frac{4}{9}cx+\frac{1}{9}c^2\right)=150$$
$$3x^2+\frac{20}{9}x^2+\frac{20}{9}cx+\frac{5}{9}c^2-150=0$$
$$\frac{47}{9}x^2+\frac{20}{9}cx+\frac{5}{9}c^2-150=0 \quad |\cdot 9$$

$$\underbrace{47}_{a}x^2+\underbrace{20c}_{b}\cdot x+\underbrace{5c^2-1350}_{c}=0$$

$$D=b^2-4ac$$
$$D=(20c)^2-4\cdot 47\cdot(5c^2-1350)$$
$$D=400c^2-940c^2+253800$$

$D=-540c^2+253800$, aby byla přímka tečnou, musí platit: $D=0$

$$-540c^2+253800=0$$
$$540c^2=253800 \quad |:540$$
$$c^2=470$$
$$\boxed{c_{1,2}=\pm\sqrt{470}}$$

---

⑧ Určete délku tětivy, kterou na elipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ vytíná přímka $3x-y+2=0$.

$$y=3x+2$$

$$\frac{x^2}{16}+\frac{(3x+2)^2}{4}=1$$
$$\frac{x^2}{16}+\frac{9x^2+12x+4}{4}=1 \quad |\cdot 16$$
$$x^2+36x^2+48x+\cancel{16}=\cancel{16}$$
$$37x^2+48x=0$$
$$x(37x+48)=0 \quad x_1=0, \quad x_2=-\frac{48}{37}$$

$$y_1=3\cdot 0+2=2 \qquad A[0;2]$$

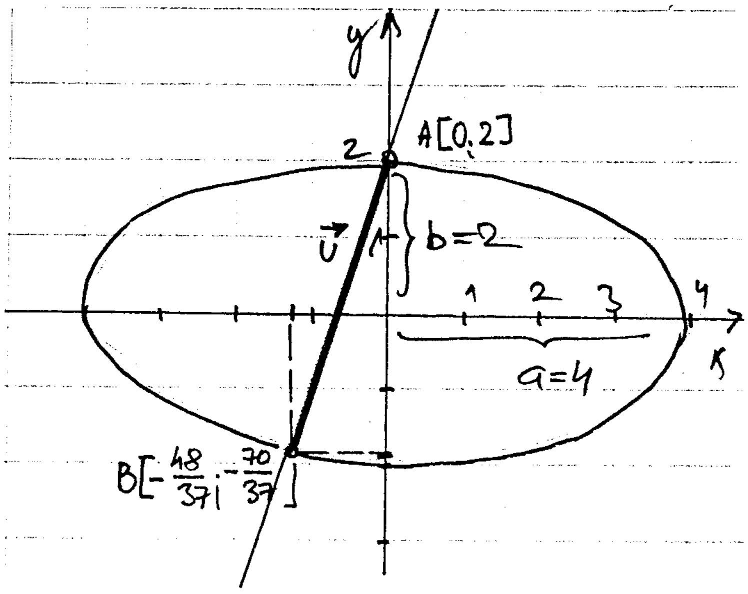
$$y_2=3\cdot\left(-\frac{48}{37}\right)+2=-\frac{144}{37}+2=-\frac{70}{37}$$

$$B\left[-\frac{48}{37};-\frac{70}{37}\right]$$

$$\vec{u}=A-B=\left(0+\frac{48}{37};2+\frac{70}{37}\right)=\left(\frac{48}{37};\frac{144}{37}\right)$$

$$|\vec{u}|=\sqrt{\left(\frac{48}{37}\right)^2+\left(\frac{144}{37}\right)^2}.$$

$$|\vec{u}|=\sqrt{\frac{2304}{1369}+\frac{20736}{1369}}=\text{další ch.}$$



$$|\vec{u}| = \sqrt{\frac{23040}{1369}}$$

$$|\vec{u}| = \frac{\sqrt{2304 \cdot 10}}{\sqrt{1369}}$$

$$|\vec{u}| = \boxed{\frac{48 \cdot \sqrt{10}}{37} \doteq 4{,}1}$$ je délka tětivy AB dané elipsy.

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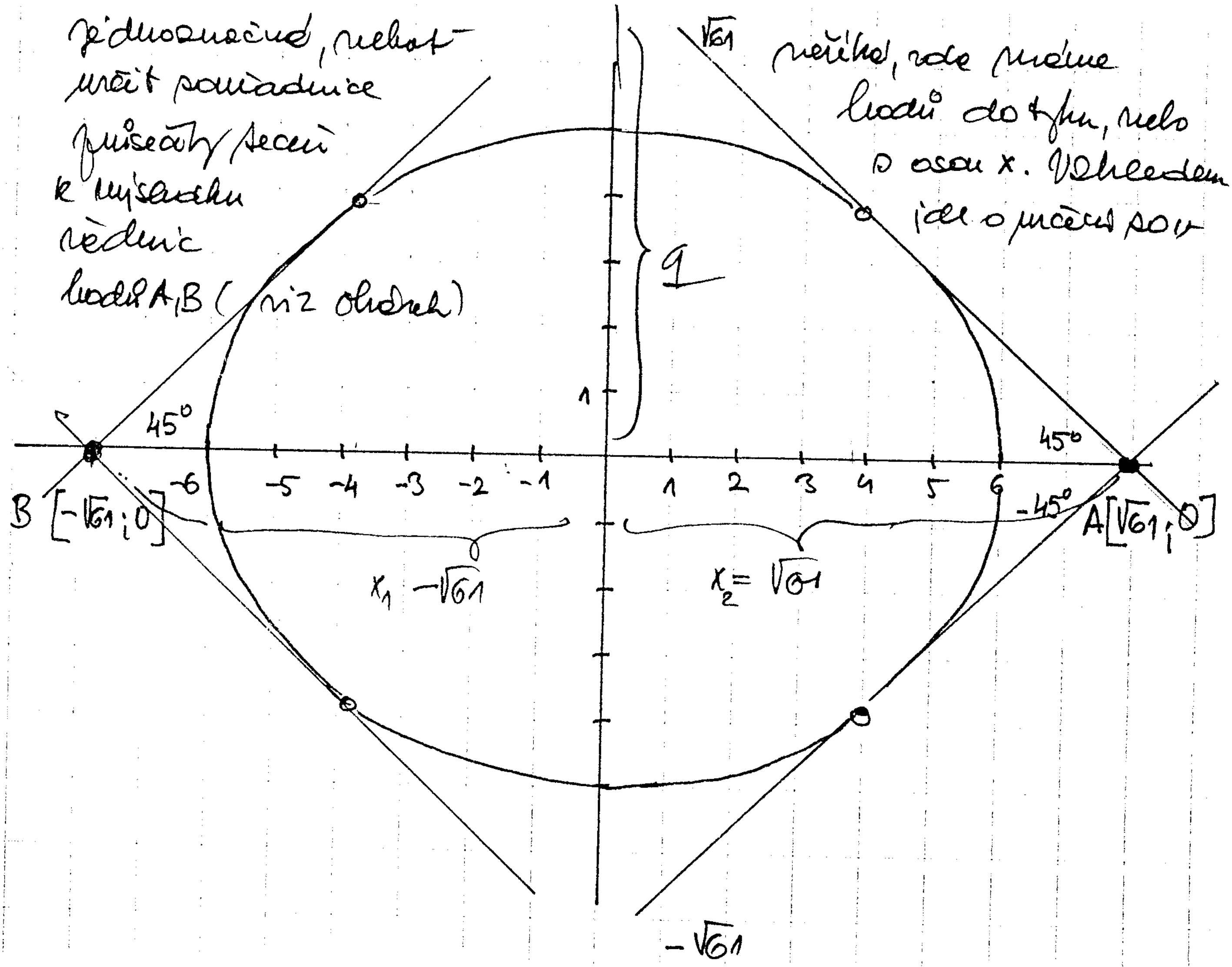
⑨ K elipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ veďte tečnu/tečny, které svírají s osou O_x úhel 45°. (Tato formulace není v pořádku

je dvojznačná, nelze určit souřadnice průsečíků tečen k myšleným řešením bodů A,B (viz obrázek)

neříká, zda máme bodů dotyku, nebo s osou x. Vzhledem jde o průsečík sou-



Tečny mají směrnice 1, nebo −1, mají tvar

$y = kx + q \qquad y = x + q$ nebo $y = -x + q$

Platí tedy.

$$\frac{x^2}{36} + \frac{(x+q)^2}{25} = 1$$

$$\frac{x^2}{36} + \frac{x^2 + 2qx + q^2}{25} = 1 \quad | \cdot 900$$

$$25x^2 + 36x^2 + 72qx + 36q^2 = 900$$

$$\underbrace{61}_{a}x^2 + \underbrace{72q}_{b}x + \underbrace{36q^2 - 900}_{c} = 0$$

$$b^2 - 4ac = 0$$

$$(72q)^2 - 4 \cdot 61 \cdot (36q^2 - 900) = 0$$

$$5184q^2 - 8784q^2 + 219600 = 0$$

$$3600q^2 = 219600$$

$$q^2 = 61$$

$$q_{1,2} = \boxed{\pm\sqrt{61}}$$

Tečny protínají osu x v bodech $A[\sqrt{61};0]$, $B[-\sqrt{61};0]$, ale též osu y v bodech $\sqrt{61}; -\sqrt{61}$, obě tečny svírají pod úhlem 90°.

---

⑩ Napište rovnici tečny k elipse $\frac{x^2}{15} + \frac{y^2}{9} = 1$, která je rovnoběžná s přímkou $2x + y - 7 = 0$

$$2x + y - 7 = 0$$

$$y = \textcircled{-2}x + 7$$

$\downarrow k = -2$ je

též směrnice tečny:

$y = kx + q \ldots y = -2x + q$

$$\frac{x^2}{15} + \frac{(-2x+q)^2}{9} = 1 \quad | \cdot 45$$

$$3x^2 + 5(-2x+q)^2 - 45 = 0$$

$$3x^2 + 5(4x^2 + 4qx + q^2) - 45 = 0$$

$$3x^2 + 20x^2 + 20qx + 5q^2 - 45 = 0$$

$$\underbrace{23}_{a}x^2 + \underbrace{20q}_{b}x + \underbrace{5q^2 - 45}_{c} = 0$$

$$D = b^2 - 4ac = (20q)^2 - 4 \cdot 23 \cdot (5q^2 - 45)$$

$$D = 400q^2 - 460q^2 + 4140 = -60q^2 + 4140$$

$$-60q^2 + 4140 = 0$$

$$60q^2 = 4140$$

$$q^2 = 69 \rightarrow q_{1,2} = \pm\sqrt{69}$$

$$y = -2x + q$$

$$2x + y - q = 0$$

$$2x + y - (\pm\sqrt{69}) = 0$$

t_1: $\boxed{2x + y + \sqrt{69} = 0}$

t_2: $\boxed{2x + y - \sqrt{69} = 0}$

---

⑪ Napište rovnici tečny k elipse $\frac{x^2}{36} + \frac{y^2}{29} = 1$, která je kolmá k přímce $2x + 3y - 11 = 0$.

$2x + 3y - 11 = 0$

$3y = -2x + 11$

$y = \boxed{-\frac{2}{3}}x + \frac{11}{3}$

$\downarrow$

je směrnice dané přímky; $-\frac{1}{k}$ bude směrnice tečny

$y = -\frac{1}{k}x + q$

$-\frac{1}{k} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}$

$y = \frac{3}{2}x + q$, dosaď ⊛ do rovnice elipsy

$$\frac{x^2}{36} + \frac{\left(\frac{3}{2}x + q\right)^2}{29} = 1$$

$$\frac{x^2}{36} + \frac{\frac{9}{4}x^2 + 3qx + q^2}{29} = 1 \quad |\cdot 36\cdot 29$$

$$29x^2 + 36\left(\frac{9}{4}x^2 + 3qx + q^2\right) = 1044$$

$$29x^2 + 81x^2 + 108qx + 36q^2 - 1044 = 0$$

$$\underbrace{110}_{a}x^2 + \underbrace{108q}_{b}\cdot x + \underbrace{36q^2 - 1044}_{c} = 0$$

$$D = (108q)^2 - 4\cdot 110\cdot(36q^2 - 1044)$$

$$D = \underbrace{11664q^2 - 15840q^2 + 459360}_{=0}$$

$$-4176q^2 + 459360 = 0$$

$$q^2 = \frac{459360}{4176}$$

$$q^2 = 110$$

$q_{1,2} = \pm\sqrt{110}$ dej do ⊛

$$y = \frac{3}{2}x \pm \sqrt{110} \quad |\cdot 2$$

$$2y = 3x \pm 2\cdot\sqrt{110}$$

$$2y = 3x \pm \sqrt{440} \rightarrow 3x - 2y \pm \sqrt{440} = 0$$

$$\boxed{t_1:\ 3x - 2y + \sqrt{440} = 0} \quad \boxed{t_2:\ 3x - 2y - \sqrt{440} = 0}$$

---

* ⑫ Určete číslo $\underline{a}$ tak, aby přímka $ax - 2y + 14 = 0$ byla tečnou elipsy $16x^2 + 25y^2 = 400$

$ax - 2y + 14 = 0$

$2y = ax + 14$

$y = \frac{a}{2}x + 7$

$$16x^2 + 25\left(\frac{a}{2}x + 7\right)^2 = 400$$

$$16x^2 + 25\cdot\left(\frac{a^2}{4}x^2 + 7ax + 49\right) - 400 = 0$$

$$16x^2 + \frac{25a^2}{4}x^2 + 175ax + 1225 - 400 = 0$$

$$\underbrace{\left(16 + \frac{25a^2}{4}\right)}_{a}\cdot x^2 + \underbrace{175a}_{b}\cdot x + \underbrace{825}_{c} = 0$$

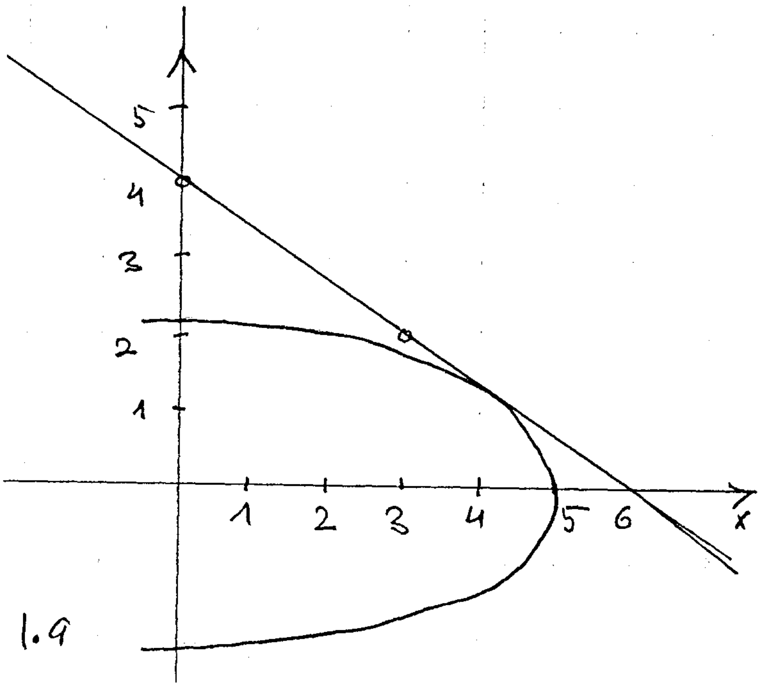
$$D = b^2 - 4ac = (175a)^2 - 4\cdot\left(16 + \frac{25a^2}{4}\right)\cdot 825$$

$$D = 30625a^2 + (-64 - 25a^2)\cdot 825 = 30625a^2 - 52800 - 20625a^2$$

$$= 10\,000a^2 - 52\,800 \ \dots\ 10\,000a^2 = 52800 \ \dots\ a^2 = \frac{528}{} \ \dots\ a_{1,2} = \boxed{\pm\sqrt{5{,}28}}$$

*⑬ Určte $\underline{b}$ tak, aby priamka $2x+3y-12=0$ bola dotyčnicou elipsy $\frac{x^2}{25}+\frac{y^2}{b^2}=1$

$$2x+3y-12=0$$
$$3y=-2x+12$$
$$y=-\frac{2}{3}x+4$$



$$\frac{X}{25}+\frac{\left(-\frac{2}{3}x+4\right)^2}{b^2}=1$$

$$\frac{x^2}{25}+\frac{\frac{4}{9}x^2-\frac{16}{3}x+16}{b^2}=1 \quad |\cdot 25b^2$$

$$b^2x^2+25\left(\frac{4}{9}x^2+\frac{16}{3}x+16\right)=25b^2$$

$$b^2x^2+\frac{100}{9}x^2-\frac{400}{3}x+400-25b^2=0 \quad |\cdot 9$$

$$9b^2x^2+100x^2-1200x+3600-225b^2=0$$

$$\underbrace{(9b^2+100)}_{a}\cdot x^2-\underbrace{1200}_{b}x\underbrace{+3600-225b^2}_{c}=0$$

$$D=b^2-4ac=(-1200)^2-4\cdot(9b^2+100)\cdot(3600-225b^2)=0$$

$$1\,440\,000-4\cdot(32\,400b^2+360\,000-2\,025b^4-22500b^2)=0$$

$$\cancel{1\,440\,000}-129\,600b^2-\cancel{1\,440\,000}+8100b^4+90\,000b^2=0$$

$$8100b^4-39\,600b^2=0 \quad |:900$$

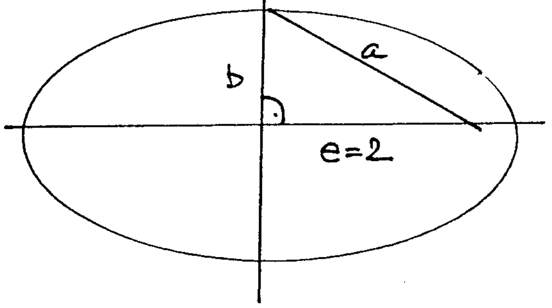
$$9b^4-44b^2=0$$

$$b^2(9b^2-44)=$$

$b^2=0$ (nevyhovuje)

$$9b^2-44=0$$
$$9b^2=44$$
$$b=\sqrt{\frac{44}{9}}$$
$$b=\frac{\sqrt{44}}{\sqrt{9}} \quad \ldots \quad \boxed{b=\frac{\sqrt{44}}{3}}$$

*⑭ Napište rovnici elipsy v osovém tvaru, je-li dána její excentricita $e=2$ a tečna $2x+3y+9=0$.



$a^2=b^2+4$

$b^2=a^2-4$

$2x+3y+9=0$

$3y=-2x-9$

$y=-\frac{2}{3}x-3$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{\left(-\frac{2}{3}x-3\right)^2}{a^2-4}=1 \quad |\cdot a^2(a^2-4)$$

$$x^2(a^2-4)+a^2\left(\frac{4}{9}x^2+4x+9\right)=a^4-4a^2$$

$$a^2x^2-4x^2+\frac{4}{9}a^2x^2+4a^2x+9a^2-a^4+4a^2=0$$

$$\frac{13}{9}a^2x^2-4x^2+4a^2x+9a^2-a^4+4a^2=0$$

$$\frac{13}{9}a^2x-4x^2+4a^2x+13a^2-a^4=0$$

$$\underbrace{\left(\frac{13}{9}a^2-4\right)}_{a}\cdot x^2+\underbrace{4a^2}_{b}x+\underbrace{13a^2-a^4}_{c}=0$$

$$D=(4a^2)^2-4\cdot\left(\frac{13}{9}a^2-4\right)\cdot(13a^2-a^4)=0$$

$$16a^4-4\cdot\left(\frac{169}{9}a^4-52a^2-\frac{13}{9}a^6+4a^4\right)=0$$

$$\cancel{16a^4}-\frac{676}{9}a^4+208a^2+\frac{52}{9}a^6-\cancel{16a^4}=0$$

$$\frac{52}{9}a^6-\frac{676}{9}a^4+208a^2=0 \quad |:a^2$$

$$\frac{52}{9}a^4-\frac{676}{9}a^2+208=0 \quad |\cdot 9$$

$$52a^4-676a^2+1872=0$$

Substituce: $a^2=m$

$$52m^2-676m+1872=0$$

$$m_{1,2}=\frac{676\pm\sqrt{67600}}{104}$$

$$m_{1,2}=\frac{676\pm 260}{104}=$$

$m_1=9 \Rightarrow a^2=9$

$m_2=4 \Rightarrow a^2=4$

$b^2=a^2-4$

$b^2=9-4$

$b^2=5$

$b^2=4-4$

$b^2=0$ nevyhovuje

Rovnice elipsy: $\boxed{\frac{x^2}{9}+\frac{y^2}{5}=1}$

KONEC ČLÁNKU 4.7