Elipsa II.

Př.1. Ukažte, že rce $x^2+4y^2-6x+32y+48=0$ je OR elipsy. Určete střed, vrcholy, ohniska.

ST rce E → doplnění na □

$$(x-3)^2-9+4\cdot(y^2+8y)+48=0$$
$$(x-3)^2-9+4\cdot(y+4)^2-4\cdot 16+48=0$$
$$(x-3)^2+4(y+4)^2=25 \quad /:25$$
$$\frac{(x-3)^2}{25}+\frac{4(y+4)^2}{25}=1$$
$$\frac{(x-3)^2}{25}+\frac{(y+4)^2}{\frac{25}{4}}=1$$

$E: \frac{(x-m)^2}{a^2}+\frac{(y-n)^2}{b^2}=1$

$S[3;-4]$ $\quad a=\sqrt{25}=5$ $\quad b=\sqrt{\frac{25}{4}}=\frac{5}{2}$

$a>b$ $\quad e^2=a^2-b^2=25-\frac{25}{4}=\frac{75}{4}$

$e=\frac{\sqrt{75}}{2}=\frac{5\sqrt{3}}{2}$

$F[m-e;n]$ $\quad F\left[3-\frac{5\sqrt{3}}{2};-4\right]$ $\quad G\left[3+\frac{5\sqrt{3}}{2};-4\right]$

$G[m+e;n]$

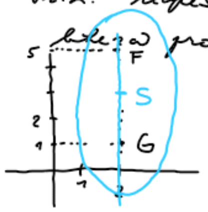
$A_1[m-a;n]$ $\quad A_1[-2;-4]$

$A_2[m+a;n]$ $\quad A_2[8;-4]$

$B_1[m;n+b]$ $\quad B_1\left[3;-\frac{3}{2}\right]$

$B_2[m;n-b]$ $\quad B_2\left[3;-\frac{13}{2}\right]$

Př.2. Napište rci E s ohnisky $F[2;5]$, $G[2;1]$, která prochází bodem $M[5;1]$



$S=S_{FG}$ $\quad S[2;3]$ $\quad e=|FS|=2$

$$\frac{(x-2)^2}{a^2}+\frac{(y-3)^2}{b^2}=1$$

$a<b$

$e^2=b^2-a^2$

$b^2=4+a^2$

$$M:\quad \frac{(5-2)^2}{a^2}+\frac{(1-3)^2}{4+a^2}=1$$
$$\frac{9}{a^2}+\frac{4}{4+a^2}=1 \quad /\cdot a^2(4+a^2)$$
$$9(4+a^2)+4\cancel{a^2}=a^4+\cancel{4a^2}$$
$$a^4-9a^2-36=0$$
$$q^2-9q-36=0$$

subst. $q=a^2$

$q_{1,2}=\left\langle \begin{matrix}12\\ -3 \times\end{matrix}\right.$

$a^2=12$

$a=\sqrt{12}=2\cdot\sqrt{3}$

$b^2=4+12=16$ $\quad b=4$

$$E:\quad \frac{(x-2)^2}{12}+\frac{(y-3)^2}{16}=1$$