

Elipsa a přímka I.

1) $p \cap E = \emptyset$... vnější přímka $D < 0$

2) $p \cap E = \{T\}$... tečna $D = 0$

3) $p \cap E = \{R_1, R_2\}$... secna $D > 0$

ne tečny elipsy v bodě $T[x_0, y_0]$

$$\frac{(x-m)(x_0-m)}{a^2} + \frac{(y-n)(y_0-n)}{b^2} = 1$$

OR tečny: $px_0x + qy_0y + r(x+x_0) + p(y+y_0) + k = 0$

Pr. 1. Určete průsečíky přímky $p: 4x + 5y = 140$ s
 $E: \frac{x^2}{625} + \frac{y^2}{400} = 1$ a napište ne tečen
 v těchto bodech.

$$p: y = -\frac{4}{5}x + 28$$

$$p \cap E: \frac{x^2}{625} + \frac{\left(-\frac{4}{5}x + 28\right)^2}{400} = 1 \quad / \cdot 625 \cdot 400$$

$$400x^2 + 625 \left(\frac{16}{25}x^2 - \frac{224}{5}x + 784 \right) = 250000$$

$$400x^2 + 400x^2 - 28000x + 490000 - 250000 = 0$$

$$800x^2 - 28000x + 240000 = 0$$

$$8x^2 - 280x + 2400 = 0$$

$$x^2 - 35x + 300 = 0$$

$$x_{1/2} = \frac{35 \pm \sqrt{(-35)^2 - 4 \cdot 300}}{2} \begin{cases} x_1 = 20 & y_1 = 12 \\ x_2 = 15 & y_2 = 16 \end{cases}$$

$$R_1 [20; 12] \quad R_2 [15; 16]$$

$$S [0; 0]$$

$$L_{1,2}: \frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$$

$$L_2: \frac{15x}{625} + \frac{16y}{400} = 1$$

$$L_1: \frac{20x}{625} + \frac{12y}{400} = 1$$

$$L_2: \underline{\underline{3x + 5y - 125 = 0}}$$

$$L_1: \underline{\underline{16x + 15y - 500 = 0}}$$

Pr. 2. Ukážete, že bod $H[6|-2]$ je bodem elipsy
 $\frac{(x-3)^2}{25} + \frac{4(y+4)^2}{25} = 1$ a napište mi rovnici
 v bodě H.

$$? H \in E? \quad \frac{(6-3)^2}{25} + \frac{4(-2+4)^2}{25} = 1$$

$$\frac{9}{25} + \frac{16}{25} = 1 \quad \frac{25}{25} = 1 \Rightarrow \underline{\underline{H \in E.}}$$

$$* : \frac{(x-3) \cdot (6-3)}{25} + \frac{4(y+4)(-2+4)}{25} = 1$$

$$3x - 9 + 8y + 32 = 25$$

$$* : \underline{\underline{3x + 8y - 2 = 0}}$$

Pr. 3. Určete c tak, aby přímka $y = x + c$ byla
 tečnou elipsy $\frac{x^2}{4} + y^2 = 1$

$$p \cap E = \{T\} \quad D = 0$$

$$x^2 + 4(x+c)^2 = 4$$

$$x^2 + 4x^2 + 8cx + 4c^2 - 4 = 0$$

$$5x^2 + 8cx + 4c^2 - 4 = 0$$

$$A = 5 \quad B = 8c \quad C = 4c^2 - 4$$

$$B^2 - 4AC = 0$$

$$64c^2 - 20(4c^2 - 4) = 0$$

$$64c^2 - 80c^2 + 80 = 0$$

$$16c^2 = 80$$

$$c^2 = \frac{80}{16} = 5$$

$$|c| = \sqrt{5}$$

$$\underline{\underline{c = \pm\sqrt{5}}}$$

Podm. $A_1: y = x + \sqrt{5}$
 $A_2: y = x - \sqrt{5}$