Elipsa a přímka II.

8X.3 Určete $c \in \mathbb{R}$, aby přímka $y = x + c$ byla tečnou

$E: \frac{x^2}{4} + y^2 = 1$

$5x^2 + 8cx + 4c^2 - 4 = 0$

tečna $\Leftrightarrow D = 0$

$A = 5 \quad B = 8c \quad C = 4c^2 - 4$

$D = B^2 - 4AC = 0$

$\frac{x^2}{4} + (x+c)^2 = 1$

$64c^2 - 20(4c^2 - 4) = 0$

$\frac{x^2}{4} + x^2 + 2cx + c^2 = 1 \quad /\cdot 4$

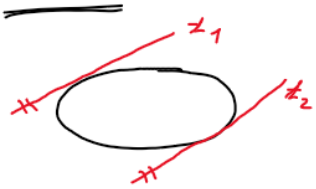
$64c^2 - 80c^2 + 80 = 0$

$x^2 + 4x^2 + 8cx + 4c^2 = 4$

$c^2 = 5$

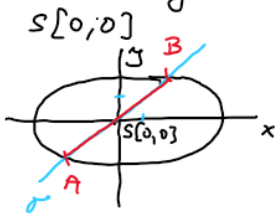
$c = \pm\sqrt{5}$

$t_1: y = x + \sqrt{5} \quad t_2: y = x - \sqrt{5}$



Sbírka:

52/5.47 Vypočtěte délku tětivy elipsy $x^2 + 2y^2 = 18$, která leží na ose I. a III. kvadr.



osa I. a III. kv. $y = x$

$|AB| = ?$

$p \cap E: \quad x^2 + 2x^2 = 18$

$3x^2 = 18 \quad x^2 = 6$

$x_1 = \sqrt{6} \quad y_1 = \sqrt{6}$

$x_2 = -\sqrt{6} \quad y_2 = -\sqrt{6}$

$A[-\sqrt{6}; -\sqrt{6}]$

$B[\sqrt{6}; \sqrt{6}]$

$|AB| = \sqrt{(2\sqrt{6})^2 + (2\sqrt{6})^2} = \sqrt{48} = 4\cdot\sqrt{3}$

52/51 Určete hodnoty $q \in \mathbb{R}$, pro které má přímka $y = x + q$ s $E: 9x^2 + 16y^2 = 144$ alespoň 1 společný bod.

$D \geq 0$

$1024q^2 - 1600q^2 + 14400 \geq 0$

$9x^2 + 16(x+q)^2 - 144 = 0$

$576q^2 \leq 14400$

$9x^2 + 16x^2 + 32qx + 16q^2 - 144 = 0$

$q^2 \leq 25$

$25x^2 + 32qx + 16q^2 - 144 = 0$

$|q| \leq 5$

$A = 25 \quad B = 32q \quad C = 16q^2 - 144$

$q \in \langle -5; 5 \rangle$

53/5.56 Elipsa je dána rovnicí $\frac{x^2}{4} + \frac{y^2}{3} = 1$

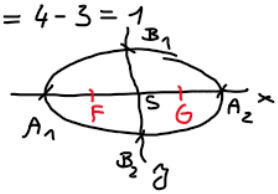
a) $a, b, e, S, A_1, A_2, B_1, B_2$

$a^2 = 4 \quad a = 2$

$e^2 = a^2 - b^2 = 4 - 3 = 1$

$b^2 = 3 \quad b = \sqrt{3}$

$e = 1$



$S[0;0]$

$A_1[-2;0] \quad A_2[2;0]$

$B_1[0;\sqrt{3}] \quad B_2[0;-\sqrt{3}]$

b) $F[-1;0] \quad G[1;0]$

c) $\varepsilon = \frac{e}{a} = \frac{1}{2}$