

Hyperbola II. - procvičování

PK. Osy H. jsou totožné s osami soust. souřadnic, $e=5$, $M[4, 3]$ leží na H. Najděte její a asymptoty.

I. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ nebo II. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (za 26)

ad I. $e^2 = a^2 + b^2$ $M \in H.$
 $a^2 + b^2 = 25$ $\frac{16}{a^2} - \frac{9}{b^2} = 1$

$b^2 = 25 - a^2$ \rightarrow soust. ric

$\frac{16}{a^2} - \frac{9}{25 - a^2} = 1$

$16(25 - a^2) - 9a^2 = a^2(25 - a^2)$

$400 - 16a^2 - 9a^2 = 25a^2 - a^4$

$a^4 - 50a^2 + 400 = 0$

Sub. $c = a^2$ $c^2 - 50c + 400 = 0$

$c_{1/2} = \frac{50 \pm \sqrt{2500 - 1600}}{2} = \frac{50 \pm 30}{2}$
 $\left. \begin{array}{l} 40 \Rightarrow a^2 = 40 \times \cancel{\emptyset} \\ 10 \Rightarrow a^2 = 10 \end{array} \right\}$

$b^2 = 25 - a^2 = 25 - 40 < 0$ mlae

$b^2 = 25 - 10 = 15$

$a_1, a_2: y = \pm \frac{b}{a} x$

$H_1: \frac{x^2}{10} - \frac{y^2}{15} = 1$

$y = \pm \sqrt{\frac{15}{10}} x = \pm \sqrt{1.5} x$

58/5.95 with parameter H: $9x^2 - 16y^2 = 144$

$\Rightarrow ? a, b, e, F, G, S, A_1, A_2 ?$

$9x^2 - 16y^2 = 144 \quad | :144$
 $\frac{x^2}{\frac{144}{9}} - \frac{y^2}{\frac{144}{16}} = 1$
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$S[0; 0]$
 $a^2 = \frac{144}{9} \Rightarrow a = \frac{12}{3} = 4$

$b^2 = \frac{144}{16} \Rightarrow b = \frac{12}{4} = 3$

$e^2 = a^2 + b^2 = 16 + 9 = 25$ $e = 5$

$F[-5; 0]$ $A_1[-4; 0]$

$G[+5; 0]$ $A_2[+4; 0]$

58/5.96 with coordinate bodies $A[6; y]$ a $C[x; \frac{3}{2}]$

leží, aby ležely na H: $9x^2 - 36y^2 = 324$

A: $9 \cdot 36 - 36y^2 = 324$ C: $9x^2 - 36 \cdot \frac{9}{4} = 324$

$36y^2 = 0$

$y = 0$

$9x^2 = 405$

$x^2 = 45 \quad |x| = \sqrt{45} = 3\sqrt{5}$

$A[6; 0]$

$C_1[3\sqrt{5}; \frac{3}{2}]$ $C_2[-3\sqrt{5}; \frac{3}{2}]$

58/5.97 with see asymptote H: $\frac{x^2}{36} - \frac{y^2}{25} = 1$

$S[0; 0] \Rightarrow a_1, a_2: y = \pm \frac{b}{a} x$

$a^2 = 36$ $b^2 = 25$

$a = 6$ $b = 5$

$a_1, a_2: y = \pm \frac{5}{6} x$

SP/100 k) write F, G, e, a_1, a_2 pro H:

$$S[0;0] \quad \frac{x^2}{4} - \frac{y^2}{3} = 1$$

$$\begin{aligned} a^2 &= 4 & b^2 &= 3 & e^2 &= a^2 + b^2 \\ a &= 2 & b &= \sqrt{3} & e^2 &= 4 + 3 = 7 \\ & & & & e &= \sqrt{7} \end{aligned}$$

$$F[-\sqrt{7}; 0] \quad G[\sqrt{7}; 0]$$

$$a_1, a_2: y = \pm \frac{a}{b} x$$

$$y = \pm \frac{\sqrt{3}}{2} x$$

SP/5.101 k) write S, A_1, A_2, F, G hyperboly

$$25x^2 - 16y^2 - 150x + 224y - 959 = 0$$

$$\square: 25(x^2 - 6x) - 16(y^2 - 14y) - 959 = 0$$

$$25(x-3)^2 - 9 \cdot 25 - 16(y-7)^2 + 16 \cdot 49 - 959 = 0$$

$$25(x-3)^2 - 16(y-7)^2 = 400 \quad | :400$$

$$I. \quad \frac{(x-3)^2}{\frac{400}{25}} - \frac{(y-7)^2}{\frac{400}{16}} = 1 \quad S[3;7]$$

$$a^2 = \frac{400}{25} \quad a = \frac{20}{5} = 4$$

$$b^2 = \frac{400}{16} \quad b = \frac{20}{4} = 5$$

$$e^2 = a^2 + b^2 = 16 + 25 = 41 \quad e = \sqrt{41}$$

$$F[3 - \sqrt{41}; 7]$$

$$G[3 + \sqrt{41}; 7]$$

$$A_1[-1; 7]$$

$$A_2[7; 7]$$