

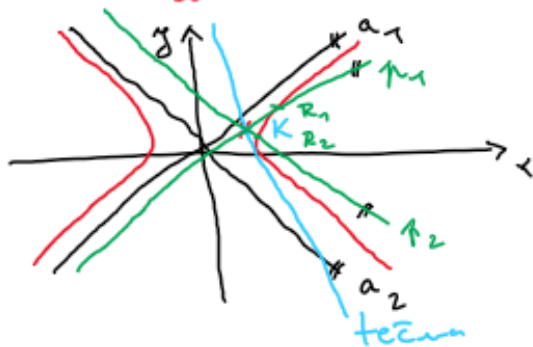
Hyperbola III. - tečna hyperboly

$T[x_0, y_0]$... tečný bod

$S[m, n]$

$$I. \quad \frac{(x-m)(x_0-m)}{a^2} - \frac{(y-n)(y_0-n)}{b^2} = 1$$

$$II. \quad \frac{(y-n)(y_0-n)}{a^2} - \frac{(x-m)(x_0-m)}{b^2} = 1$$



$t_1 \parallel a_1$

$t_1 \cap H = \{R_1\}$

$t_2 \parallel a_2$

$t_2 \cap H = \{R_2\}$

Pr. 1 $K \in H$. Bodem $K[2; 3]$ hyperboly $\frac{x^2}{2} - \frac{y^2}{9} = 1$

veďte rovnici přímky, která má s H. právě 1 spol. bod.

a) tečna v bodě K $S[\overset{m}{0}; \overset{n}{0}]$
 $K \equiv T \quad K[\overset{x_0}{2}, \overset{y_0}{3}]$
 $x - \frac{y}{3} - 1 = 0$

$$\frac{x \cdot 2}{2} - \frac{y \cdot 3}{9} = 1 \quad t: \underline{\underline{3x - y - 3 = 0}}$$

b) přímky \parallel s asymptotami a_1, a_2

$$a^2 = 2 \quad a = \sqrt{2} \quad b^2 = 9 \quad b = 3$$

$$a_{1,2}: y = \pm \frac{3}{\sqrt{2}} x = \pm \frac{3\sqrt{2}}{2} x$$

$$\left. \begin{array}{l} t_1 \parallel a_1 \\ K \in t_1 \end{array} \right\} \begin{array}{l} y = kx + q \\ y = \frac{3\sqrt{2}}{2} x + q \end{array}$$

$$K: 3 = \frac{3\sqrt{2}}{2} \cdot 2 + q \quad q = 3 - 3\sqrt{2}$$

$$t_1: \underline{\underline{y = \frac{3\sqrt{2}}{2} x + 3 - 3\sqrt{2}}}$$

$$\left. \begin{array}{l} t_2 \parallel a_2 \\ K \in t_2 \end{array} \right\} \begin{array}{l} y = -\frac{3\sqrt{2}}{2} x + q \\ q = 3 + 3\sqrt{2} \end{array}$$

$$K: 3 = -\frac{3\sqrt{2}}{2} \cdot 2 + q \quad t_2: \underline{\underline{y = -\frac{3\sqrt{2}}{2} x + 3 + 3\sqrt{2}}}$$

Pr. 2 najděte tečny H. $2x^2 - y^2 = 2$, které jsou rovnoběžné s přímkou $\pi: y = 2x$.

H.: $2x^2 - y^2 = 2 \quad | :2$

$$\frac{x^2}{1} - \frac{y^2}{2} = 1$$

$\star \parallel \pi$

$\star: y = 2x + q$

T: $y_0 = 2x_0 + q$

? $T[x_0, y_0]$? $\Rightarrow \frac{x \cdot x_0}{1} - \frac{y \cdot y_0}{2} = 1$

$T \in H: 2x_0^2 - y_0^2 = 2$

řešit syst. rovnic s parametrem q

$$2x_0^2 - (2x_0 + q)^2 = 2$$

$$2x_0^2 - (4x_0^2 + 4x_0q + q^2) = 2$$

$$-2x_0^2 - 4qx_0 - q^2 - 2 = 0$$

$$2x_0^2 + 4qx_0 + q^2 + 2 = 0 \Rightarrow \text{kvadratic. rov. s param. } q$$

$A = 2$

tečna $\Leftrightarrow D = 0 \Leftrightarrow B^2 - 4AC = 0$

$B = 4q$

$$16q^2 - 8(q^2 + 2) = 0$$

$C = q^2 + 2$

$$8q^2 = 16$$

$$q^2 = 2 \quad q = \pm\sqrt{2}$$

$\star: y = 2x + q$

$\star_1: y = 2x + \sqrt{2}$

Pozu. $T[x_0, y_0] = ?$

$T_1 \in \star_1 \cap H.$

$\star_2: y = 2x - \sqrt{2}$

$T_2 \in \star_2 \cap H.$

Syst. rovnic