Sbírka úloh VII.

32/76 Ukažte, že $p \parallel q$ a určete jejich vzdálenost

a) $p: 3x-4y+15=0$ $\quad q: 6x-8y=0$

$\vec{n}_p = (3;-4)$

$\vec{n}_q = (6;-8) = (3;-4)$ $\Big\} p \parallel q$

$$d = \frac{|3\cdot 4 - 4\cdot 3 + 15|}{\sqrt{9+16}} = \frac{15}{5} = 3j$$

$$d = \frac{|a\cdot a_1 + b a_2 + c|}{\sqrt{a^2+b^2}}$$

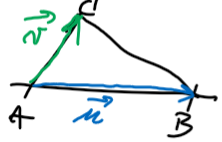
a, b, c ... souř. přímky p

A: $x = 4$ ... vzorec

$-8y = -24 \quad y = 3$

32/79 $S_{\Delta ABC} = ?$ $\quad A[-1;2], B[3;5], C[4;-3]$

$$S_\Delta = \frac{|\vec{u} \times \vec{v}|}{2}$$



$\vec{u} = \overrightarrow{AB} = (4;3)$

$\vec{v} = \overrightarrow{AC} = (5;-5)$

$\vec{u} = (4;3;0)$

$\vec{v} = (5;-5;0)$

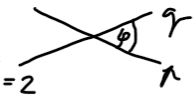
$\vec{u} \times \vec{v} = (0;0;-35)$

$S_\Delta = \frac{35}{2} j^2 = 17{,}5\, j^2$

33/84 odchylka přímek $p: x-3=0$ a $q: x\sqrt{3} - y + 5 = 0$

$\vec{n}_p = (1;0)$ $\quad |\vec{n}_p| = 1$

$\vec{n}_q = (\sqrt{3};-1)$ $\quad |\vec{n}_q| = \sqrt{3+1} = 2$



$$\cos\varphi = \frac{|\vec{n}_p \cdot \vec{n}_q|}{|\vec{n}_p| \cdot |\vec{n}_q|}$$

$$\cos\varphi = \frac{\sqrt{3}}{1\cdot 2} = \frac{\sqrt{3}}{2} \Rightarrow \varphi = 30°$$

33/88 Bodem $M[1;3]$ veďte přímku, která svírá s přímkou $p: 4y-5=0$ úhel o velikosti 45°.

$$\cos\varphi = \frac{|\vec{n}_p \cdot \vec{n}_q|}{|\vec{n}_p| \cdot |\vec{n}_q|}$$

$$\frac{\sqrt{2}}{2} = \frac{|4 n_2|}{4\cdot\sqrt{n_1^2+n_2^2}}$$

$$\frac{|n_2|}{\sqrt{n_1^2+n_2^2}} = \frac{\sqrt{2}}{2}$$

$q: n_1 x + n_2 y + c = 0$

$n_1 + 3n_2 + c = 0$

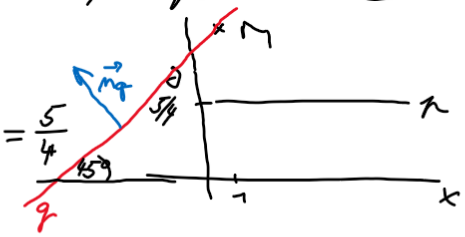
$\Rightarrow$ úloha nemá řešení ✗

$\vec{n}_p = (0;4)$ $\quad \cos 45° = \frac{\sqrt{2}}{2}$ $\quad \vec{n}_q = (n_1, n_2)$

2. způsob

$p \parallel$ osa x

$p: 4y-5=0 \Rightarrow y = \frac{5}{4}$



$y = kx + q$ $\quad k = \operatorname{tg}\varphi$

$k = \operatorname{tg} 45° = 1$

$y = x + q$

$M: 3 = 1 + q \quad q = 2$

$q: y = x + 2$

$x - y + 2 = 0$