

# AG 3D - Skriška I.

35/4.1 PV p̄mky a)  $A[5, -8, 2]$ ,  $\vec{u} = (4, 3, -1)$

$$x = 5 + 4t$$

$$y = -8 + 3t$$

$$z = 2 - t$$

35/4.2 PV p̄mky  $\pi$ ;  $A \in \pi \wedge \pi \parallel \overleftrightarrow{BC}$

a)  $A[9, -3, 1]$ ,  $B[-4, -7, 6]$   $C[2, -5, 3]$

$$\vec{u} = \overrightarrow{BC} = (6, 2, -3)$$

$$x = 9 + 6t$$

$$y = -3 + 2t$$

$$z = 1 - 3t //$$

36/4.14 PV roviny dané

a)  $A[1, 3, -1]$   $B[2, 3, 3]$   $C[-2, -5, -7]$

$$\vec{u} = \overrightarrow{AB} = (1, 0, 4)$$

$$x = 1 + t - 3s$$

$$y = 3 - 2s$$

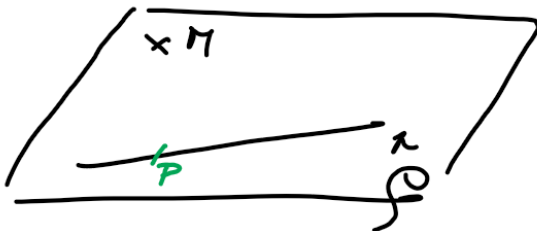
$$z = -1 + 4t - 6s //$$

37/4.18 PV roviny:

a)  $M[3, 2, -1]$  a p̄mhou:  $x = 2 - t$

$$y = 3 + 2t$$

$$z = -t$$



$P[2, 3, 0]$

$$\vec{u} = (-1, 2, -1)$$

$$\vec{v} = \overrightarrow{PM} = (1, -1, -1)$$

$$\rho: x = 3 - t + s$$

$$y = 2 + 2t - s$$

$$z = -1 - t - s //$$

AG3D - Sbrvka III.

1)  $A[1,0,1], B[-2,1,3], C[4,-1,2], D[1,1,1]$

PV roviny  $\vec{ABC}$  a m\u00edst zda  $D \in \vec{ABC}$

$\vec{u} = \vec{AB} = (-3, 1, 2)$  PV:  $x = 1 - 3t + 3s$   
 $\vec{v} = \vec{AC} = (3, -1, 1)$   $y = x - s$   
 $z = 1 + 2t + s$

D:  $1 = 1 - 3t + 3s \Rightarrow 1 = 1 - 3(1+s) + 3s$   $x = 1 - 3 - 3s + 3s$   
 $1 = t - s \Rightarrow t = 1 + s$   $y = -3$   
 $1 = 1 + 2t + s$   $z = -3$   
 $D \notin \vec{ABC}$

2) PV p\u00e1rn\u00fdch k prode\u00e1z\u00ed dole  $A[2, -1, 3]$   
 a je || o p\u00e1rn\u00fdch  $\vec{CD}, C[2, 0, 1], D[-1, 1, 1]$

$\vec{u} = \vec{CD} = (-1, 1, 0)$

$\rho: x = 2 - t$   
 $y = -1 + t$   
 $z = 3$

38/4.25 najit 3 body r rovnici dane OR:  $2x - y + 3z - 4 = 0$   
 kde rovnice jsou jednozna\u00e7n\u00e9 uspo\u00edn\u00e1ny

$A[1, 1, 1]$   $2 - 1 + 3z - 4 = 0 \Rightarrow z = 1$   
 $B[0, 1, \frac{5}{3}]$   $-1 + 3z - 4 = 0 \Rightarrow 3z = 5 \Rightarrow z = \frac{5}{3}$   
 $C[0, 0, \frac{4}{3}]$

38/4.27  $\rho: 4x - 3y - 2z + d = 0$   $A \in \rho$   $A[7, 6, -5]$   $d = ?$

A:  $4 \cdot 7 - 3 \cdot 6 - 2 \cdot (-5) + d = 0$   $d = -4$

38/4.28 OR roviny prode\u00e1z\u00ed 3 body

a)  $A[2, -1, 0], B[-1, 2, -3], C[-2, -3, 1]$

$\vec{u} = \vec{AB} = (-3, 3, -3) = (-1, 1, -1)$   
 $\vec{v} = \vec{AC} = (-4, -2, 1) = (-4, -2, 1)$   $\begin{matrix} 1 & -1 \\ -2 & 1 \end{matrix} = 1 - 2 = -1$   
 $\vec{u} \times \vec{v} = (-1, 5, 6)$

OR:  $-x + 5y + 6z + d = 0$

A:  $-2 - 5 + d = 0$   $d = 7$

$-x + 5y + 6z + 7 = 0$

38/4.29 OR roviny dane PV:  $x = 1 - t + 3s$

$y = 4 + 2t - s$   
 $z = -3 - t + s$

$\vec{u} = (-1, 2, -1)$   $A[1, 7, -3]$

$\vec{v} = (3, -1, 1)$

$\vec{u} \times \vec{v} = \vec{n} = (1, -2, -5)$

$\begin{matrix} 2 & -1 \\ -2 & 1 \end{matrix} \Rightarrow 2 - 1 = 1$

$x - 2y - 5z + d = 0$

$\rho: x - 2y - 5z - 2 = 0$

A:  $1 - 14 + 15 + d = 0$   $d = -2$

38/4.34 OR roviny  $A \in \rho$   $A[-3, 5, -4]$   $\rho \perp \vec{n}$

$\vec{n} = (1, -2, -1)$

$x - 2y - z + d = 0$  A:  $-3 - 10 + 4 + d = 0$   $d = 6$

$\rho: x - 2y - z + 6 = 0$

38/4.35 OR roviny proch. body  $A[2, -3, 1]$  a je  $\perp$  k p\u00e1rnici

$\rho: x = t$   $\vec{u} = (1, 3, -1) = \vec{n}_\rho$

$y = 2 + 3t$   
 $z = 1 - t$

$x + 3y - z + d = 0$   $\rho: x + 3y - z + 8 = 0$

A:  $2 - 9 - 1 + d = 0$

$d = 8$

AG 3D - Stránka IV.

39/4.36 OR roviny určení bodu A [2, -3, 1] a průmkou

$x = t, y = 2 + 3t, z = 1 - t$

$\vec{u} = (1, 3, -1)$

$5x + 2y + 11z + d = 0$

B [0, 2, 1]

A:  $10 - 6 + 11 + d = 0$

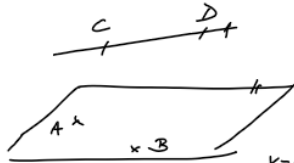
$\vec{n} = \vec{AB} = (-2, 5, 0)$

$d = -15$

$\vec{u} \times \vec{n} = \vec{m}_p = (5, 2, 11)$   $\rho: 5x + 2y + 11z - 15 = 0$

39/4.37 OR roviny proclázející body A [4, -1, 2]

B [2, 0, -1] a je || o  $\vec{CD}$  C [3, 2, -4], D [1, -1, -3]



$\vec{CD} = \vec{u} = (-2, -3, 1)$

$\vec{AB} = \vec{n} = (-2, 1, -3)$

$\vec{u} \times \vec{n} = \vec{m}_p = (8, -8, -8) = (1, -1, -1)$

$x - y - z + d = 0$

A:  $4 + 1 - 2 + d = 0 \Rightarrow d = -3$

$\rho: x - y - z - 3 = 0$

39/4.39 V2a): poloha přímek p, q:

a)  $p: x = 8 - 4t, y = 4 + 8t, z = -12t$

$q: x = 3 + 3s, y = 1 - 6s, z = -2 + 9s$

$\vec{u} = (-4, 8, -12) = (-1, 2, 3)$   $\vec{v} = (3, -6, 9) = (1, -2, 3)$

$\vec{u} = -\vec{v} \Rightarrow p \parallel q$

? jsou p a q totožné?

$P[8, 4, 0] \rightarrow q: \begin{cases} 8 = 3 + 3s \Rightarrow s = 5/3 \\ 4 = 1 - 6s \Rightarrow s = -1/2 \\ 0 = -2 + 9s \end{cases} \Rightarrow p \not\parallel q$

39/4.40 v2a): poloha p a q

$p = \vec{AB}, q = \vec{CD}$  A [2, 0, 3], B [-2, 3, 15], C [4, -1, 2], D [1, 2, -1]

$\vec{u} = \vec{AB} = (-4, 3, 12)$

$\vec{v} = \vec{CD} = (-3, 3, -9) = (-1, 1, -3)$

?  $\vec{u} = k \cdot \vec{v}$ ?

$\vec{u} \neq k \cdot \vec{v} \Rightarrow p \not\parallel q$

(řízuvoběžky nebo mimoběžky)

$p \cap q: \begin{cases} 1 - s = 2 - 4t \\ 2 + s = 3t \end{cases} *$

$-1 - 3s = 3 + 12t$

\*  $3 = 2 - t \Rightarrow t = -1$   
 $s = -5$

ověřit 3. rovnici

$-1 + 15 \neq 3 - 12 \Rightarrow$  mimoběžky

40/4.48 v2a): poloha rovin p a v

a)  $p: x = 2 + 3u - v, y = 1 - 9u + v, z = 3 - 12u - 2v$

$v: x = 1 - 2s + t, y = 2s - 3t, z = 2 - 4s - 4t$

$\vec{a} = (3, -9, -12) = (1, -3, -4)$

$\vec{b} = (-1, 1, -2)$

$\vec{m}_p = \vec{a} \times \vec{b} = (10, 6, -2) = (5, 3, -1)$

$\vec{c} = (-2, 2, -4) = (-1, 1, -2)$

$\vec{d} = (1, -3, -4)$

$\vec{m}_v = \vec{c} \times \vec{d}$

$p \parallel v$  nebo  $p \equiv v$

$R[2, 1, -5] ? R \in v ? \begin{cases} 2 = 1 - 2s + t \\ 1 = 2s - 3t \\ -5 = 2 - 4s - 4t \end{cases} \Rightarrow \begin{cases} t = -1 \\ s = -1 \end{cases}$

$-5 = 2 + 4 + 4$

$-5 \neq 10 \Rightarrow R \notin v \Rightarrow$

$p \parallel v$  (rovnoběžky)

AG3D - Sbirka IV.

40/451 uvoj: položka rovin  $\rho$  a  $\sigma$  + průsečnice

b)  $\rho: 2x - 5y + 4z - 10 = 0$      $\sigma: x - y - z - 2 = 0$

$$\left. \begin{aligned} \vec{m}_\rho &= (2, -5, 4) \\ \vec{m}_\sigma &= (1, -1, -1) \end{aligned} \right\} \vec{m}_\rho \neq k \vec{m}_\sigma \Rightarrow \rho \nparallel \sigma \Rightarrow \text{ex. průsečnice } \mu$$

$$\begin{aligned} 2x - 5y + 4z - 10 &= 0 \\ x - y - z - 2 &= 0 \end{aligned}$$

$$\begin{aligned} 2x - 5y &= 10 - 4z \\ x - y &= 2 + z \quad | \cdot (-2) \end{aligned} \quad \downarrow (+)$$

$z = t$

$$-3y = 6 - 6t \quad | : (-3)$$

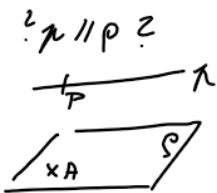
$y = -2 + 2t$

$$x = 2 + t - 2 + 2t = 3t$$

$\mu: x = 3t$   
 $y = -2 + 2t$   
 $z = t$

41/457 uvoj: položka roviny  $\rho$  a přímky  $\mu$

a)  $\rho: \begin{cases} x = 1 - 2z + 5s \\ y = 2 + 3z \\ z = 4s \end{cases}$      $\mu: \begin{cases} x = 4 - 3t \\ y = 5 - 3t \\ z = 4 - 4t \end{cases}$



$P[4, 5, 4]$   
 $? \mu \parallel \rho?$   
 $? P \in \rho?$   
 $4 = 1 - 2z + 5s \Rightarrow 4 = 1 - 2 + 5 \Rightarrow 4 = 4 \Rightarrow P \in \rho$   
 $5 = 2 + 3z \Rightarrow z = 1$   
 $4 = 4s \Rightarrow s = 1$

$z$  volim  
kdy  $Q \in \mu: Q[1, 2, 0]$      $? Q \in \rho?$   
 $z = 1$      $Q \equiv A \in \rho \Rightarrow Q \in \rho \Rightarrow \underline{\underline{\mu \subset \rho}}$

41/458 uvoj: položka  $\rho$  a  $\mu$

c)  $\rho: 2x - 7y + z - 5 = 0$      $\mu: \begin{cases} x = 4 - t \\ y = 8 - 3t \\ z = 3 + 2t \end{cases}$

$\vec{m}_\rho = (2, -7, 1)$

$\vec{u}_\mu = (-1, -3, 2)$

$\vec{m}_\rho \cdot \vec{u}_\mu = -2 + 21 + 2 = 21 \neq 0$

$? \mu \parallel \rho?$

$\vec{m}_\rho \perp \vec{u}_\mu \Rightarrow \vec{m}_\rho \cdot \vec{u}_\mu = 0$

$\Rightarrow \underline{\underline{\mu \nparallel \rho}} \quad R \in \mu \cap \rho$

41/460 uvoj: položka přímky AB a roviny  $\rho$

e)  $A[1, -4, -1]$      $\rho: x + 2y - 5z + 4 = 0$   
 $B[7, -4, 1]$

$? A \in \rho? \quad \frac{1 - 8 + 3 + 4}{0} = 0 \quad A \in \rho$   
 $? B \in \rho? \quad \frac{7 - 8 - 3 + 4}{0} = 0 \quad B \in \rho$   
 $\left. \begin{aligned} A \in \rho \\ B \in \rho \end{aligned} \right\} \underline{\underline{AB \subset \rho}}$

AG3D - Stránka V.

42/4.65 vztaj. poloha pa  $\rho$  + průsečíky

c)  $\rho: x = 7 + 5z$   
 $y = 4 + z$   
 $z = 5 + 4z$

$\rho: 3x - y + 2z - 5 = 0$   
 ?  $\rho \parallel \rho? \vec{n} \cdot \vec{m}_\rho = 0$   
 $15 - 1 + 8 = 0 \quad 22 \neq 0$

$\vec{n} = (5, 1, 4)$   
 $\vec{m}_\rho = (3, -1, 2)$

$\Rightarrow \rho \not\parallel \rho$

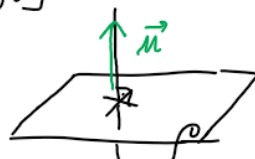
R:  $3(7+5z) - (4+z) + 2(5+4z) - 5 = 0$   
 $z = -1 \quad x = 2$   
 $y = 3$   
 $z = 1$

$R[2; 3; 1]$

42/4.67 OR roviny proložena:  $A[1, -3, 0]$

a  $\rho \perp$  na  $\rho: x = 1 + z$   
 $y = 3 + 2z$   
 $z = -7 - 4z$

$\vec{n} = (1, 2, -4) = \vec{m}_\rho$



$\rho: x + 2y - 4z + d = 0$   
 $A: 1 - 6 + 0 + d = 0 \quad d = 5$


$\rho: x + 2y - 4z + 5 = 0$

42/4.68 určete  $m \in \mathbb{R}$  tak, aby  $\vec{AB} \parallel \rho$

$A[3, -1, m], B[-2, 1, 3], \rho: 2x - 3y + z - 7 = 0$

$\vec{m}_\rho = (2, -3, 1)$   
 $\vec{AB} = (-1, 2, 3 - m)$

$\vec{AB} \perp \vec{m}_\rho \Rightarrow \vec{m}_\rho \cdot \vec{AB} = 0$   
 $2(-1) + (-3) \cdot 2 + 1 \cdot (3 - m) = 0$   
 $-8 + 3 - m = 0$   
 $m = -5$



42/4.69 vztaj. poloha roviny  $x + 2y - z + 4 = 0$  a přímky, která je proložena roviny

$\rho: 2x - y - 3z + 3 = 0, \sigma: 3x + y - 4z + 7 = 0$

$\rho \in \rho \cap \sigma \Rightarrow PV \rho:$

$2x - y - 3z + 3 = 0$	$2x - y = 3z - 3$
$3x + y - 4z + 7 = 0$	$3x + y = 4z - 7$
	$\frac{5x}{5} = \frac{7z - 10}{5}$
	$x = \frac{7}{5}z - 2$

$y = 2x - 3z + 3 = \frac{14}{5}z - 4 - 3z + 3 = -\frac{1}{5}z - 1$

$\rho: x = -2 + \frac{7}{5}z$   
 $y = -1 - \frac{1}{5}z$   
 $z = z$

$x = -2 + 7z$   
 $y = -1 - z$   
 $z = 5z$

$\tau: x + 2y - z + 4 = 0$

$\vec{n}_\tau = (1, 2, -1) \quad \vec{u}_\rho = (7, -1, 5)$

$\vec{n}_\tau \cdot \vec{u}_\rho = 7 - 2 - 5 = 0 \Rightarrow \vec{n}_\tau \perp \vec{u}_\rho \Rightarrow \tau \parallel \rho$

?  $\rho \subset \tau?$   
 $\tau: -2 - 2 + 0 + 4 = 0 \quad 0 = 0 \Rightarrow P \in \tau$   
 ka  $P[-2, -1, 0] \Rightarrow \rho \subset \tau$  (přímka leží v rovině)

AG3D - Sbírala VI

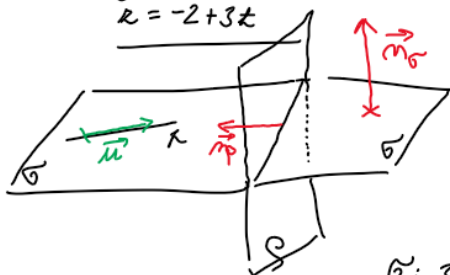
43/4.73 OR roviny:  $M \in \rho \wedge \rho \perp \vec{AB}$

a)  $M[3, -1, 6], A[3, 2, -1], B[5, -1, -3]$

$\vec{u} = \vec{AB} = (2, -3, -2) = \vec{m}_\rho$   $\rho: 2x - 3y - 2z + d = 0$   
 $M: 6 + 3 - 12 + d = 0 \Rightarrow d = 3$   
 $\rho: 2x - 3y - 2z + 3 = 0$

44/4.74 OR roviny  $\sigma: \rho \subset \sigma \wedge \sigma \perp \rho$

a)  $\rho: x = 1 + 6z$   
 $y = -3 - 3z$   
 $z = -2 + 3z$



$\vec{m}_\sigma \perp \vec{u}$   
 $\vec{m}_\sigma \perp \vec{m}_\rho$   
 $\vec{m}_\sigma = \vec{u} \times \vec{m}_\rho$

$\vec{u} = (6, -3, 3) = (2, -1, 1)$

$\vec{m}_\rho = (6, -1, 4)$

$\vec{m}_\sigma = (-3, -2, 4)$   
 $= (3, 2, -4)$

$\sigma: 3x + 2y - 4z + d = 0$

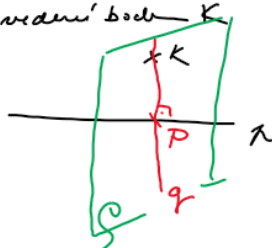
$\rho \in \sigma: 3 - 6 + 8 + d = 0 \Rightarrow d = -5$

$\sigma: 3x + 2y - 4z - 5 = 0$

$\rho: [1, -3, -2]$

44/4.78 souřadnice paty kolmice vedené bodem K a přímce  $\rho$

a)  $K[5, -4, 2]$   $\rho: x = 1 - z$   
 $y = -4 + z$   
 $z = 7 + 3z$



$\rho \perp \rho$   
 $K \in \rho$

$\vec{u} = \vec{m}_\rho = (-1, 1, 3)$   $\rho: x - y - 3z + d = 0$   
 $= (1, -1, -3)$   $K: 5 + 4 - 6 + d = 0 \Rightarrow d = -6$

$\rho: x - y - 3z - 6 = 0$

$K \rightarrow \rho: 1 - 1 + 4 - 21 - 9z - 6 = 0$

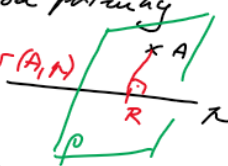
$z = -2$

$\rho: [3, -6, 1]$

45/4.89 vzdál. bodu  $A[5, -1, 3]$  od přímky  $\rho$

$\rho: x = -1 + 2z$   
 $y = -5 + 3z$   
 $z = -2 + 2z$

$|AR| = r(A, \rho)$



$\rho \perp \rho$   $A \in \rho$   $\vec{u} = \vec{m}_\rho = (2, 3, 2)$   $\rho: 2x + 3y + 2z + d = 0$

$\rho \cap \rho: -2 + 4z - 15 + 9z - 4 + 4z - 13 = 0 \Rightarrow A: 10 - 3 + 6 + d = 0 \Rightarrow d = -13$

$\rho: 2x + 3y + 2z - 13 = 0$

$R[3, 1, 2]$   $|AR| = \sqrt{(3-5)^2 + (1+1)^2 + (2-3)^2} = \sqrt{4+4+1} = 3$   
 $r(A, \rho) = 3$

45/4.92 vzdál. bodu  $A[3, 5, -6]$  od roviny  $\rho: 2x - 2y + z - 8 = 0$

$d(A, \rho) = \frac{|a \cdot a_1 + b \cdot a_2 + c \cdot a_3 + d|}{\sqrt{a^2 + b^2 + c^2}}$   $\vec{m}_\rho = (2, -2, 1)$

$d(A, \rho) = \frac{|3 \cdot 2 - 2 \cdot 5 - 6 - 8|}{\sqrt{4 + 4 + 1}} = \frac{18}{3} = 6$

AG 3D - Sbirka VII.

45/4.94 rovnice,  $\rho \parallel \sigma$  a vzdálenost  $v(\rho, \sigma)$

$$\rho: \begin{cases} x+y+z-6=0 \\ x+y+z-3=0 \end{cases} \quad \vec{m}_\rho = \vec{m}_\sigma \Rightarrow \rho \parallel \sigma$$

zvolíme  $A \in \rho: A[0; 0; 6]$

$$d(A, \sigma) = \frac{|0+0+6-3|}{\sqrt{1^2+1^2+1^2}} = \frac{3}{\sqrt{3}} = \underline{\underline{\sqrt{3} \text{ j}}}$$

45/4.95 rovnice  $\rho \parallel \sigma$ ,  $d(\rho, \sigma) = ?$

$$\rho: \begin{cases} x = 2s \\ y = 2z \\ z = 2 - z - s \quad | \cdot 2 \end{cases} \quad \sigma: \begin{cases} x = 1 - u - 2v \\ y = u \\ z = v \quad | \cdot 2 \end{cases}$$

?02  $\rho: x+y+2z-4=0$      $\sigma: x+y+2z-1=0$   
 $\vec{m}_\rho = \vec{m}_\sigma \Rightarrow$   
 $-4 \neq -1 \Rightarrow$   $\rho \parallel \sigma$

$$A \in \rho: A[0; 0; 2] \quad d(A, \sigma) = \frac{|0+0+4-1|}{\sqrt{1^2+1^2+2^2}} = \frac{3}{\sqrt{6}} = \underline{\underline{\frac{\sqrt{6}}{2} \text{ j}}}$$

46/4.97 odchylka  $\rho$  a  $\sigma$

$$a) \rho: x+y+2z-5=0$$

$$\sigma: x-2y-z+3=0$$

$$\cos \varphi = \frac{|\vec{m}_\rho \cdot \vec{m}_\sigma|}{|\vec{m}_\rho| \cdot |\vec{m}_\sigma|}$$

$$|\angle(\rho, \sigma)| = |\angle(\vec{m}_\rho, \vec{m}_\sigma)|$$

$$\vec{m}_\rho = (1, 1, 2)$$

$$\vec{m}_\sigma = (1, -2, -1)$$

$$\cos \varphi = \frac{|1-2-2|}{\sqrt{6} \cdot \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\underline{\underline{\varphi = 60^\circ}}$$

46/4.98 odchylka roviny

$$\rho: 3x+5=0$$

$$\sigma: \begin{cases} x = 3 + z - 2s \\ y = 2 - z + 2s \\ z = -1 - 4z \end{cases}$$

$$\vec{m}_\rho = (3, 0, 0)$$

$$\vec{u} = (1, -1, -4) \quad \begin{matrix} -1 & -4 \\ 2 & 0 \end{matrix}$$

$$\vec{v} = (-2, 2, 0)$$

$$\cos \varphi = \frac{|24+0+0|}{\sqrt{9} \cdot \sqrt{64+64}} = \frac{24}{3 \cdot \sqrt{128}} = \frac{8}{8 \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$= \frac{24}{2.64} = \frac{8}{2.64} = \frac{1}{\sqrt{2}} \Rightarrow \underline{\underline{\varphi = 45^\circ}}$$

$$\vec{m}_\sigma = (8; 8; 0) = \cancel{(1; 1; 0)}$$

*ide o vektory  $\vec{m}_\sigma$*

46/4.99 odchylka přímky  $\rho$  a roviny  $\sigma$

$$\rho: \begin{cases} x = 5 + t \\ y = 1 + 3t \\ z = -2t \end{cases}$$

$$\sigma: 2x - y + 3z - 4 = 0$$

$$y = 1 + 3t$$

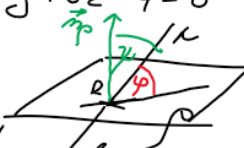
$$z = -2t$$

$$\vec{u} = (1, 3, -2)$$

$$\vec{m}_\sigma = (2, -1, 3)$$

$$\cos \psi = \frac{|2-3-6|}{\sqrt{14} \cdot \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$$

$$\psi = 60^\circ \quad \varphi = 90^\circ - 60^\circ = \underline{\underline{30^\circ}}$$



AG3D - Sbírka VIII.

46/4.100 je dáno:  $\rho, \sigma, \rho$

a)  $|\angle \rho, \rho| = ?$  b)  $|\angle \rho, \sigma| = ?$  c)  $|\angle \rho, \sigma|$

$$\begin{array}{l} \rho: \quad x = 1 + t \\ \quad y = 2 - t \\ \quad z = t \end{array} \quad \rho: \quad 3y + z = 0 \quad \sigma: \quad \begin{array}{l} x = 5 - t - 3s \\ y = 1 + t - 3s \\ z = 3 + 4t \end{array}$$

a)  $\vec{u} = (1, -1, 1)$   $\cos \psi = \frac{|-3|}{\sqrt{3} \cdot 3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \psi = 54^\circ 45'$   
 $\vec{m}_\rho = (0, 3, 0)$   
 $\varphi = 90^\circ - 54^\circ 45' = 35^\circ 15'$

b)  $\vec{a} = (-1, 1, 4) = (-1, 1, 4)$   
 $\vec{b} = (-3, -3, 0) = (1, 1, 0)$   
 $\vec{a} \times \vec{b} = \vec{m}_\sigma = (12, -12, 6) = (2, -2, 1)$

$$\cos \psi = \frac{|12 + 12 + 6|}{\sqrt{3} \cdot \sqrt{324}} = \frac{30}{\sqrt{3} \cdot 18} = \frac{5}{\sqrt{3} \cdot 3} = \frac{5}{3\sqrt{3}}$$

$$\psi = 15^\circ 47' \quad \varphi = 90^\circ - 15^\circ 47' = 74^\circ 13'$$

kontrola:  $\vec{m}_\sigma = (-4, 4, -2) = (2, -2, 1)$

$$\cos \psi = \frac{|2 + 2 + 1|}{\sqrt{3} \cdot 3} = \frac{5}{\sqrt{3} \cdot 3} \Rightarrow \varphi = 74^\circ 13'$$

c)  $\vec{m}_\rho = (0, 3, 0)$   $\cos \varphi = \frac{|0 - 6 + 0|}{\sqrt{3^2} \cdot \sqrt{9}} = \frac{6}{9} = \frac{2}{3}$   
 $\vec{m}_\sigma = (2, -2, 1)$

$$\varphi = 48^\circ 11'$$

46/4.102 je dáno:  $\rho, \rho$

a)  $|\angle \rho, \rho| = ?$  b)  $\rho \cap \rho$

$$\begin{array}{l} \rho: \quad x - y + z + 1 = 0 \\ \quad x + y + 3z - 3 = 0 \end{array} \quad \rho: \quad x - y + z = 0$$

$\Rightarrow ? PV:$

$$z = t$$

$$\begin{array}{l} x - y = -1 - t \\ x + y = 3 - 3t \end{array}$$

$$2x = 2 - 4t \quad 1 - 2t + 1 + t = t$$

$$\boxed{x = 1 - 2t} \quad \boxed{y = 2 - t}$$

a)  $\vec{u} = (-2, -1, 1)$

$$\vec{m}_\rho = (1, -1, 1)$$

$$\cos \psi = \frac{|-2 + 1 + 1|}{\sqrt{6} \cdot \sqrt{3}} = \frac{0}{\sqrt{18}} = 0$$

$$\psi = 90^\circ \Rightarrow \varphi = 0^\circ$$

b)  $P[1, 2, 0]$

?  $P \in \rho$ ?  $1 - 2 + 0 \neq 0$

$P \notin \rho \Rightarrow$

$\rho \parallel \rho \Rightarrow \underline{\underline{\rho \cap \rho = \emptyset}}$

