GNB – 3. čtvrtletní práce – 1A, 3C

Hodnocení: 1: 10-9b, 2: 8-7b, 3: 6-5b, 4: 4-3b, 5: 2-1b.

A

Jméno: autor

Třída:

1. Řešte v R: $\frac{2x+8}{x+3}+\frac{1}{x}\geq 2$
2. Řešte v R: $\left|\frac{5+7x}{x}\right|>\frac{1}{2}$
3. Řešte soustavu rovnic:

$$3x+2y-z=8$$
$$2x-2y+2z=0$$
$$7-2y+3z=8$$

4. Určete definiční obor funkce $y=\sqrt{\frac{x}{x-3}-\frac{2x}{x+4}}$
5. Uprav a zjednoduš, urči podmínky: $\dfrac{\frac{x^2+9x+14}{x^2-x-12}}{\frac{x^2+6x-7}{x^2-2x-15}}$

1) $\frac{2x+8}{x+3}+\frac{1}{x}-2\geq 0$

$\frac{(2x+8)x+x+3-2x(x+3)}{x\cdot(x+3)}\geq 0$

$\frac{2x^2+8x+x+3-2x^2-6x}{x\cdot(x+3)}\geq 0$

$\frac{3x+3}{x(x+3)}\geq 0$

$\frac{3(x+1)}{x(x+3)}\geq 0$

N.B. $-1;\ 0;\ -3$

$<\ (\ (\ $

	$(-\infty;-3)$ (−4)	$(-3;-1\rangle$ (−2)	$\langle -1;0)$ (−0,5)	$(0;+\infty)$ (1)
$x+1$	−	−	+	+
x	−	−	−	+
$x+3$	−	+	+	+
$\frac{()}{()()}$	−	(+)	−	(+)

$x\in(-3;-1\rangle\cup(0;+\infty)$

5) $\dfrac{\frac{x^2+9x+14}{x^2-x-12}}{\frac{x^2+6x-7}{x^2-2x-15}}=\dfrac{\frac{(x+2)(x+7)}{(x-4)(x+3)}}{\frac{(x-1)(x+7)}{(x-5)(x+3)}}=\dfrac{(x+2)(x-5)}{(x-1)(x-4)}$

$x\neq 4;-3;1;-7;5$

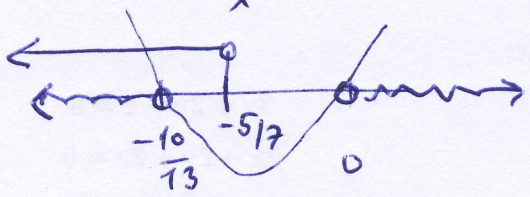
2) $\left|\frac{5+7x}{x}\right| > \frac{1}{2}$ $\quad x \neq 0$

$\frac{|5+7x|}{|x|} > \frac{1}{2}$ $\quad$ N.B. $-\frac{5}{7}$; 0

	$(-\infty; -5/7)$	$(-5/7; 0)$	$(0; +\infty)$
$5+7x$	−	+	+
x	−	−	+

I. $\frac{-5-7x}{-x} > \frac{1}{2} \Rightarrow \frac{2(7x+5)}{x} > 1 \Rightarrow \frac{13x+10}{x} > 0$

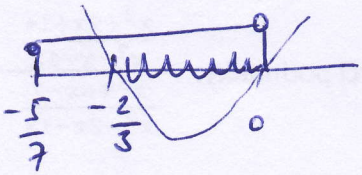
N.B. $-\frac{10}{13}$; 0



$x_1 \in (-\infty; -\frac{10}{13})$

II. $\frac{5+7x}{-x} > \frac{1}{2} \Rightarrow 1 + \frac{10+14x}{x} < 0 \Rightarrow \frac{15x+10}{x} < 0$

N.B. $\frac{-10}{15} = -\frac{2}{3}$; 0



$x_2 \in (-\frac{2}{3}; 0)$

III. $\frac{5+7x}{x} > \frac{1}{2} \Rightarrow \frac{13x+10}{x} > 0 \Rightarrow x_3 \in (0; +\infty)$

$K = (-\infty; -\frac{10}{13}) \cup (-\frac{2}{3}; 0) \cup (0; +\infty)$

3) I. $3x + 2y - z = 8$
II. $x - y + z = 0$ $\quad / \cdot (-3)$ $\quad \oplus$
III. $-2y + 3z = 1$

$5y - 4z = 8 \quad / \cdot 3$
$-2y + 3z = 1 \quad / \cdot 4$ $\quad \oplus$

$15y - 12z = 24$
$-8y + 12z = 4$

$7y = 28$

$y = 4$

III. $-2 \cdot 4 + 3z = 1$
$3z = 9$
$z = 3$

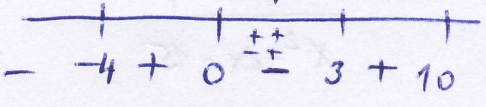
II. $x - 4 + 3 = 0$
$x = 1$

$[1; 4; 3]$

4) $\frac{x(x+4) - 2x(x-3)}{(x-3)(x+4)} \geq 0$ $\quad x \neq 3; -4$

$\frac{10x - x^2}{(x-3)(x+4)} \geq 0 \Rightarrow \frac{x(10-x)}{(x-3)(x+4)} \geq 0$

N.B. 0; 10; −4; 3



$D_f = \langle -4; 0 \rangle \cup \langle 3; 10 \rangle$

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B

Jméno: autor

Třída:

1. Řešte v R: $\frac{2x}{2x-4} - \frac{2}{x+1} \geq 1$
2. Řešte v R: $1 - |1 - 2x| = |2x - 4| - |x|$
3. Řešte soustavu rovnic:

$$3x + 2y - z = 8$$
$$2x - 2y + 2z = 2$$
$$-4x + y - z = -7$$

4. Určete definiční obor výrazu a zjednodušte:

$$\frac{x^2 - 5x + 6}{x^2 - 4x + 4}$$

5. Určete definiční obor funkce: $y = \sqrt{x^2 - 7x + 12} - \sqrt{x - 1}$

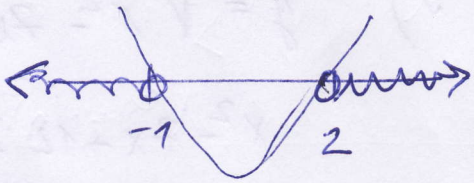
$\frac{2x}{2(x-2)}$

1) $\frac{x(x+1) - 2(x-2) - (x-2)(x+1)}{(x-2)(x+1)} \geq 0$

$\frac{x^2 + x - 2x + 4 - x^2 - x + 2}{(x-2)(x+1)} \geq 0$

$\frac{6}{(x-2)(x+1)} \geq 0$

$(x-2)(x+1) > 0$



$x \in (-\infty; -1) \cup (2; +\infty)$

2) N.B. $\frac{1}{2}; 2; 0$

	$(-\infty; 0)$	$\langle 0; \frac{1}{2})$	$\langle \frac{1}{2}; 2)$	$\langle 2; +\infty)$
$1-2x$	+	+	−	−
$2x-4$	−	−	−	+
x	−	+	+	+

I. $1 - (1-2x) = (-2x+4) - (-x)$

$2x = -x + 4$

$3x = 4$

$x = 4/3 \quad K_1 = \emptyset$

II. $1 - (1-2x) = (-2x+4) - x$

$2x = -3x + 4$

$5x = 4$

$x = \frac{4}{5} \notin \langle 0; \frac{1}{2})$

$K_2 = \emptyset$

III. $1 - (-1+2x) = -2x + 4 - x$

$2 - 2x = -2x + 4 - x$

$x = 2 \in \langle \frac{1}{2}; 2 \rangle \quad K_3 = \{2\}$

IV. $1 - (-1+2x) = 2x - 4 - x$

$2 - 2x = x - 4$

$-3x = -6$

$x = 2 \in \langle 2; +\infty)$

$K_4 = \{2$

$K = \{2\}$

3) I. $3x + 2y - z = 8$
II. $x - y + z = 2$
III. $-4x + y - z = -7$

I+II.: $4x + y = 10$
II+III: $-3x = -5 \Rightarrow x = \frac{5}{3}$

$4 \cdot \frac{5}{3} + y = 10 \quad y = 10 - \frac{20}{3} = \frac{10}{3}$

z II: $z = 2 - x + y$

$z = 2 - \frac{5}{3} + \frac{10}{3} = 2 + \frac{5}{3} = \frac{11}{3}$

$\left[\frac{5}{3}; \frac{10}{3}; \frac{11}{3}\right]$

4) $\frac{x^2 - 5x + 6}{x^2 - 4x + 4} = \frac{(x-3)(x-2)}{(x-2)^2} = \frac{x-3}{x-2}$

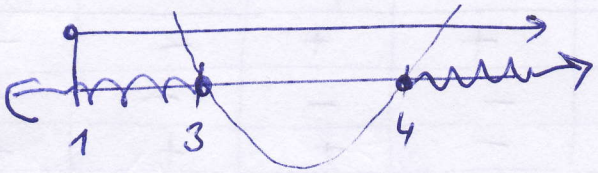
$D_f = \mathbb{R} - \{2\}$

5) $y = \sqrt{x^2 - 7x + 12} + \sqrt{x-1}$

$x^2 - 7x + 12 \geq 0 \quad \wedge \quad x - 1 \geq 0$

$x_{1/2} = \frac{7 \pm \sqrt{49 - 48}}{2} \begin{matrix} ① \\ ; \end{matrix} \begin{matrix} 4 \\ 3 \end{matrix}$

$(x-4)(x-3) \geq 0 \quad \wedge \quad x \geq 1$



$x_2 \in \langle 1; +\infty)$

$x_1 \in (-\infty; 3\rangle \cup \langle 4; +\infty)$

$D_f: x \in x_1 \cap x_2$

$D_f = \langle 1; 3\rangle \cup \langle 4; +\infty)$

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Hodnocení: 1: 10-9b, 2: 8-7b, 3: 6-5b, 4: 4-3b, 5: 2-1b.

C

Jméno: Anton

Třída:

1. Řešte v R: $\frac{x+3}{x-3}+\frac{x-1}{x-5}=4$
2. Řešte v R: $2-|3-x|+|x|<2x-|3x-1|$
3. Řešte soustavu rovnic:

   $x^2+y^2-4=0$

   $x+2y=4$
4. Určete definiční obor funkce: $y=\sqrt{\frac{x^2-5x+6}{x^2-4x+4}}$
5. Rozložte na součin kvadratické trojčleny:
   a) x^2+x-12
   b) $3x^2-36x+105$

1) podm. $x \neq 3; 5$

$(x+3)(x-5)+(x-1)(x-3)=4(x-3)(x-5)$

$2x^2-6x-12=4x^2-32x+60$

$2x^2-26x+72=0$

$x^2-13x+36=0$

$x_{1,2}=\frac{+13\pm\sqrt{169-144}}{2}$ (5) $\begin{matrix}/\ +4\\ \backslash\ +9\end{matrix}$

$K=\{+9;+4\}$

2) N.B. $3; 0; \frac{1}{3}$

	$(-\infty;0\rangle$	$\langle 0;\frac{1}{3}\rangle$	$\langle\frac{1}{3};3\rangle$	$\langle 3;+\infty)$
$3-x$	+	+	+	−
x	−	+	+	+
$3x-1$	−	−	+	+

I. $2-(3-x)-x<2x-(-3x+1)$

$-1<5x-1$

$5x>0$

$x>0$ $K_1=\emptyset$

II. $2-(3-x)+x<2x-(-3x+1)$

$-1+2x<5x-1$

$3x>0$

$x>0$ $K_2=\langle 0;\frac{1}{3}\rangle$

III. $2-(3-x)+x<2x-(3x-1)$

$-1+2x<2x-3x+1$

$3x<2$

$x<\frac{2}{3}$ $K_3=\langle\frac{1}{3};\frac{2}{3}\rangle$

IV. $2-(-3+x)+x<2x-(3x-1)$

$5<2x-3x+1$

$x<-4$ $K_4=\emptyset$

$K=K_2\cup K_3=\langle 0;\frac{2}{3}\rangle$

3) $x^2+y^2=4$

$x+2y=4 \Rightarrow x=4-2y$

$(4-2y)^2+y^2=4$

$16-16y+4y^2+y^2=4$

$5y^2-16y+12=0$

$y_{1/2}=\frac{16\pm\sqrt{256-240}}{10}$ ④

$y_1=2$

$y_2=\frac{6}{5}$

$[0;2]$

$\left[\frac{8}{5};\frac{6}{5}\right]$

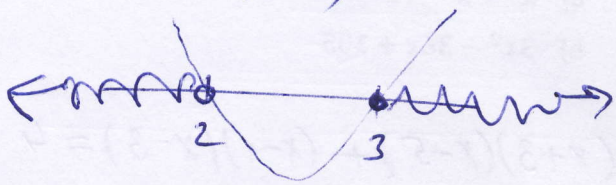
16
16
96
16
256

$x_1=4-2\cdot 2=0$ $\quad x_2=4-2\cdot\frac{6}{5}=4-\frac{12}{5}=\frac{8}{5}$

4) $\frac{x^2-5x+6}{x^2-4x+4}\geq 0 \Rightarrow \frac{(x-3)(x-2)}{(x-2)^2}\geq 0 \quad x\neq 2$

W.B. 3; 2

$\frac{x-3}{x-2}\geq 0$



$D_f=(-\infty;2)\cup(3;+\infty)$

5) a) $x^2+x-12=(x-3)(x+4)$

Viet. vz. $x_1\cdot x_2=-12$ $\quad 3\cdot(-4)$ ✓

$x_1+x_2=-1$ $\quad (-3)\cdot 4$ ×

$x_1=3 \quad x_2=-4$

h) $3x^2-36x+105=3\cdot(x-5)(x-7)$

$x_{1/2}=\frac{+36\pm\sqrt{(-36)^2-12\cdot 105}}{6}$ $\quad 5$, 7

$3\cdot(x^2-12x+35)$

$\frac{12\pm\sqrt{144-140}}{2}$ ② $\quad 7$, 5